informs
http://pubsonline.informs.org/journal/mnsc/

MANAGEMENT SCIENCE
Articles in Advance, pp. 1–17
ISSN 0025-1909 (print), ISSN 1526-5501 (online)

Testing by Competitors in Enforcement of Product Standards

Erica L. Plambeck,[a] **Terry A. Taylor**[b]

[a] Graduate School of Business, Stanford University, Stanford, California 94305; [b] Haas School of Business, University of California, Berkeley, Berkeley, California 94720

Contact: elp@stanford.edu, http://orcid.org/0000-0001-9378-5190 (ELP); taylor@haas.berkeley.edu, http://orcid.org/0000-0002-1488-8923 (TAA)

Received: July 8, 2015
Revised: November 8, 2016; September 28, 2017
Accepted: October 10, 2017
Published Online in Articles in Advance: August 6, 2018

https://doi.org/10.1287/mnsc.2017.3023

Copyright: © 2018 INFORMS

Abstract. Firms have an incentive to test competitors' products to reveal violations of safety and environmental standards, in order to have competitors' products blocked from sale. This paper shows that testing by a regulator crowds out testing by competitors, and can reduce firms' efforts to comply with the product standard. Relying on competitor testing (i.e., having the regulator test only to verify evidence of violations provided by competitors) is most effective in large or concentrated markets in which firms have strong brands and high quality, and for standards that are highly valued by consumers. Under those conditions, firms tend to test competitors' products and exert high compliance effort. Conversely, unless compliance is highly valued by consumers, a firm with low quality does not draw testing from competitors, and so does not comply. Enforcing a product standard through competitor testing encourages entry by such low-quality, noncompliant firms and can reduce quality investment by incumbents. Stripping offending products of labels (such as "Energy Star"), instead of blocking them from the market, eliminates the problem of entry by low-quality, noncompliant firms, but may reduce incumbents' compliance efforts.

History: Accepted by Martin Lariviere, operations management.
Funding: The research was supported by the National Science Foundation [Grant 0239840].
Supplemental Material: The online supplement is available at https://doi.org/10.1287/mnsc.2017.3023.

Keywords: manufacturing • environment • regulation • policy

1. Introduction

This paper derives insights from a game-theoretic model of firms' efforts to comply with a product standard and to test competitors' products for violations. Each firm chooses how much cost to incur to increase the likelihood that its product complies with the standard. Compliance is a "credence" attribute (Darby and Karni 1973): in the normal course of use, consumers do not directly observe whether a product complies with the standard. However, testing by a competitor or a regulator may detect a violation, causing a product to be blocked from the market (or alternatively, in Section 4.4, stripped of a label signifying compliance with a voluntary standard).

The paper is motivated by the potentially important role of firms' testing of competitors' products in the enforcement of product standards, such as those regulating energy efficiency, hazardous substances and safety. Historically, the United States and European Union (EU) have relied to a large extent on testing by competitors in enforcement of energy efficiency performance standards (Wiel and McMahon 2005, Gaffigan 2007, Department of Energy 2010). Since 2002, the European Union has restricted the use of increasingly many hazardous substances in an increasingly wide variety of products. In addition, the European Union has required products to be labeled with the manufacturer's identity, which promotes testing and reporting by competitors, and helps regulators prevent the sale of products that violate the restrictions on hazardous substances or other safety or environmental standards. These regulations have substantially increased the frequency of products being blocked from EU markets due to safety violations, to more than 20,000 such measures during 2004–2015 (Croft and Strongman 2004; Kapoor 2012; European Commission 2012, 2015). In 2002, authorities blocked the sale of Sony PlayStation consoles due to a violation of the European Union's new Restrictions on Hazardous Substances in Electronics (RoHS), reputedly in response to a tip from one of Sony's competitors about the cadmium in a peripheral cable (Hess 2006), causing Sony to miss \$110 million in revenue (Shah and Sullivan 2002). Since then, testing by competitors has become increasingly common in the EU consumer electronics industry, according to Green Supply Line (2006) and Smith (2008). In the United States and European Union, some consumer product manufacturers test competitors' products and report violations of safety standards to have competitors' products blocked from those markets (Overfelt 2006, Ross 2007), though data on firms'

RIGHTSLINK

reporting of competitors' violations are not publicly available.[1]

For government regulatory authorities, testing to detect a violation is costly and difficult because there are many potential failure modes (specific ways in which a product might fail to meet a standard) to be tested. For example, to determine whether or not one personal computer is RoHS-compliant would require disassembly and testing of approximately 3,000 constituent materials for each of six restricted substances, which would cost regulatory authorities as much as $200,000 (Bruschia 2008).[2] As a second example, U.S. energy efficiency standards require a product to have power consumption below specified thresholds in a variety of different operating modes and ambient conditions. Additional failure modes—not addressed in those detailed specifications—can cause a violation due to insufficient efficiency in actual use.[3]

In contrast, when a firm identifies *how* a competitor's product violates a standard and reports that information to the regulator, the regulator can cheaply and reliably verify that the product is noncompliant (Bruschia 2008, Smith 2008). Regulators follow up on credible reports that specify precisely how a product violates the standard and are well-supported by testing data (Bruschia 2008, Smith 2008). Why aren't regulators flooded with false or nonspecific reports of violations? Only a correct, specific report enables the regulator to verify noncompliance. Providing fraudulent information to a regulatory authority in order to damage a competitor is illegal in the European Union, United States, and many other countries; in the United States, for example, the penalties for doing so include fines and imprisonment for up to five years (see subsection 18 of U.S. Code Section 1001(a)). Therefore, in the model in this paper, each firm decides how much to spend on testing each competitor's product. A firm submits evidence to the regulator that a competitor's product is noncompliant if and only if its testing produces that evidence. The evidence characterizes the mode by which the product fails to meet the standard and, upon obtaining that evidence, the regulator verifies that the product is noncompliant and prevents its sale.

In testing a product to detect a violation of a product standard, competitors tend to be more efficient than the regulator, for the following reasons. Firms typically have equipment and trained staff for testing the compliance of their own products, so can test competitors' products at little additional cost (Wiel and McMahon 2005, Gaffigan 2007, Smith 2008). They often purchase competitors' products to evaluate other quality characteristics, which reduces their procurement cost to test compliance (Day 2007). Through their own compliance efforts, firms develop a better understanding than the regulator of when and how a competitor's product is likely to be noncompliant, so can better target their testing efforts (Hess 2006). For example, a firm's understanding of a competitor's suppliers' reputation and capabilities can provide insight into which components or constituent materials are likely to cause noncompliance. Unlike firms, government regulatory authorities lack a profit motive for efficiency in testing, and their budget for testing must be raised through taxes that distort the economy and reduce social welfare (Polinsky 1980).

This paper addresses standards for credence attributes that consumers are willing to pay for, and ones they are not. Credence attributes with private benefits for consumers include, for example, restrictions on toxic and endocrine-disrupting substances in children's products, and energy efficiency. (Energy efficiency is a credence attribute because, though a consumer may look at a monthly household electricity bill, she cannot easily evaluate the energy efficiency of an individual product (Ko and Simons 2016).) Other product standards cover credence attributes with a social or environmental benefit, for which consumers might be unwilling to pay a premium. For example, RoHS aims to mitigate the environmental impact of electronics waste, and many survey respondents indicate they will not pay more for such "green" electronics (Saphores et al. 2007).

1.1. Literature

Risk that a product is defective (violates a standard) arises from operational challenges, notably the difficulties of supply chain management to ensure that all components of a product meet design specification. Conformance-quality (i.e., product conformance to design specification) effort and inspection effort are key operational decisions that reduce the probability a product is defective. An extensive operations management literature addresses variants of conformance-quality and inspection effort, e.g., statistical process control (Porteus and Angelus 1997) and contractual incentives for suppliers in conjunction with inspection of suppliers' output (Baiman et al. 2000, Balachandran and Radhakrishnan 2005, Babich and Tang 2012), auditing of suppliers' conformance-quality capability (Hwang et al. 2006), or investment in suppliers' conformance-quality capability (Zhu et al. 2007). "Compliance" effort in this paper represents all the effort that a firm exerts to reduce the probability its product violates a standard, including conformance-quality and inspection effort. In addition, this paper extends the operations management literature—which focuses on inspection of one's own or one's suppliers' products—by incorporating inspection of competitors' products. The model in this paper is similar to those in Baiman et al. (2000), Balachandran and Radhakrishnan (2005), Hwang et al. (2006), and Babich and

RIGHTSLINK

Tang (2012) in that a firm's entire output is either defective or conforming and, in the former event, inspection effort (testing expenditure) increases the probability of detecting the defect.

Papers surveyed in Cohen (1999) model a regulator choosing effort to detect a violation, in game theoretic equilibrium with a potential violator. Those papers adopt a variety of assumptions regarding the regulator's objective function and whether or not the regulator moves first by committing to a detection-effort level. For example, in Mookherjee and Png (1992), the regulator moves first and maximizes social welfare, whereas in Boyer et al. (2000) the regulator moves simultaneously with the potential violator and maximizes the regulator's own utility (the expected fine less cost of detection effort). Rather than restrict attention to one of those various formulations, this paper treats testing by the regulator as a parameter for sensitivity analysis. One may interpret that parameter as an initial commitment by the regulator or as the level of regulator testing anticipated by the firms. The paper identifies conditions under which relying on competitor testing (having the regulator do zero testing to detect violations, only verify reports of violations) is effective in enforcing a product standard. Similarly, Mookherjee and Png (1992) show that a regulator should only verify reports of violations, in a setting in which a violator chooses the severity of his violation, a victim reports a violation with high probability, and the cost of verification is low.

Whereas much of the literature on regulation or voluntary standards for products' credence attributes assumes perfect monitoring and compliance (see Roe and Sheldon 2007, Heyes and Martin 2017, and papers surveyed therein), as notable exceptions, Mason (2011) emphasizes that tests for credence quality are noisy, McCluskey and Loureiro (2005) treat testing by a regulator as a parameter for sensitivity analysis, and Feddersen and Gilligan (2001) consider an exogenous probability of monitoring by an activist. To the best of our knowledge, this paper is the first to address firms' testing of credence attributes in competitors' products.

The literature on whistleblowing models whistleblowing by firms in cartels (see Spagnolo 2008, Bigoni et al. 2012, and papers surveyed therein) or by employees (Austen-Smith and Feddersen 2008, Ting 2008). Whereas that literature focuses on a potential whistleblower's decision *whether or not to report* a violation to a regulator, this paper focuses on firms' *efforts to detect* competitors' violations; in the setting of this paper, reporting a detected violation is optimal. The whistleblowing literature and this paper are consistent in assuming that any report of a violation is correct, and the regulator acts in response to a report.

In Li and Peeters (2017), a firm decides whether to test the quality of a competitor's product and report that information to *consumers*. Their results are discussed in Section 4.4, which also addresses reporting to consumers.

1.2. Overview of Main Results

The first part of Section 3 shows that testing by a regulator crowds out testing by competitors and can reduce compliance. Specifically, a low level of testing by the regulator fails to increase firms' compliance efforts or the detection probability for a noncompliant product (i.e., the probability that, in the event that a product violates the standard, the regulator or a competitor detects that violation). It simply causes the firms to do less testing. A high level of testing by the regulator causes firms not to test, and can strictly reduce all firms' compliance efforts. These results, combined with the observation that firms can detect violations in competitors' products at lower cost and more effectively than a regulator, suggest that social welfare might be improved by relying on competitor testing (having the regulator do zero testing to detect violations).

Therefore, the second part of Section 3 identifies conditions under which relying on competitor testing is effective in enforcing a product standard. When the regulator does not test, competitor testing occurs if and only if at least two competing firms have sufficiently high product quality. That quality threshold is low—competitor testing tends to occur—in a large or concentrated market. With symmetric firms, the detection probability for a noncompliant product initially increases and then decreases with the number of competitors, and also is nonmonotonic in the market size and a firm's quality. Nevertheless, each firm's compliance effort decreases with the number of competitors, increases with the size of the market, and increases with its product quality.

Section 4 shows that those results largely hold in extensions of the model with consumers forming rational expectations about the likelihood that a product is compliant, endogenous qualities and quantities, fines, fixed costs of testing, and duplication in firms' testing activities. Furthermore, Section 4 builds on Section 3 by providing guidance for a regulator that relies on competitor testing.

In particular, Section 4 provides insight into whether a regulator should strip offending products of labels (such as "Energy Star"), instead of blocking them from the market. The blocking penalty enforced through competitor testing increases the expected profit of firms with low quality because they do not draw testing from competitors and benefit when their competitors are blocked from the market. Consequently, blocking encourages entry by low-quality, noncompliant firms. Switching to the labeling penalty eliminates such entry, but reduces incumbents' compliance efforts.

Section 5 explains how the results apply to settings with third-party testing.

RIGHTSLINK

2. Model

N firms compete in a market of size m governed by a product standard. Each firm $i \in \mathcal{N} \equiv \{1, .., N\}$ makes an effort $e_i \in [0,1]$ to comply with the standard and incurs a cost $c_i(e_i)$ that is positive, strictly increasing, and convex. Firm i produces mq_i units of its product, which is compliant with probability e_i. Then each firm i tests the product of competitor-firm j at level $t_{ij} \geq 0$, for $j \in \mathcal{N} \backslash i$, and incurs a cost $\Sigma_{j \in \mathcal{N} \backslash j} t_{ij}$. The regulator tests the product of firm i at level $t_{Ri} \geq 0$ for $i \in \mathcal{N}$, and incurs a cost $\Sigma_{i \in \mathcal{N}} t_{Ri}$. Let $\mathbf{t}_i = \langle t_{1i}, t_{2i}, .., t_{i-1,i}, t_{i+1,i}, .., t_{Ni}, t_{Ri} \rangle$ denote the vector of testing levels. If firm i's product is noncompliant, then with probability $d_i(\mathbf{t}_i)$ testing provides evidence of this noncompliance to at least one party that tested firm i at a strictly positive level. A firm submits evidence to the regulator that a competitor's product is noncompliant if and only if its testing produces that evidence. The evidence characterizes the mode by which the product fails to meet the standard and, upon obtaining that evidence, the regulator confirms that the product is noncompliant and prevents its sale. Thus, with probability

$$s_i(e_i, \mathbf{t}_i) \equiv 1 - d_i(\mathbf{t}_i)(1 - e_i)$$

firm i successfully brings its product to market; with probability $d_i(\mathbf{t}_i)(1 - e_i)$ firm i sells nothing. That is, the sales quantity for firm $i \in \mathcal{N}$ is the random variable $m\underline{q}_i$ with

$$\underline{q}_i \equiv \begin{cases} q_i & \text{with probability } s_i(e_i, \mathbf{t}_i), \\ 0 & \text{with probability } 1 - s_i(e_i, \mathbf{t}_i). \end{cases}$$

We assume that for a given vector of the firms' compliance and testing efforts, the $\underline{q}_i$ and $\underline{q}_j$ for $j \neq i$ are independent. The detection probability for a noncompliant product $d_i(\mathbf{t}_i)$ is componentwise strictly increasing, continuously differentiable and satisfies $d_i(0, \ldots, 0) = 0$ and $\lim_{t_{ji} \to \infty} d_i(\mathbf{t}_i) = \bar{d}$ for $j \in \{R, \mathcal{N} \backslash i\}$ where $\bar{d} \in (0,1)$. Furthermore, to the extent that the detection probability for firm i is already high, additional testing is less effective: for $j \in \mathcal{N} \cup R$ and $i \in \mathcal{N} \backslash j$,

$$(\partial/\partial t_{ji}) d_i(\mathbf{t}_i) < (\partial/\partial t_{ji}) d_i(\mathbf{t}'_i), \quad \text{for } \mathbf{t}_i, \mathbf{t}'_i \in R_+^{N+1} \text{ such that } d_i(\mathbf{t}_i) > d_i(\mathbf{t}'_i). \quad (1)$$

As in the workhorse model of vertically differentiated quality used in Motta (1993) and references therein, the product of firm i has "quality" u_i, and the mass m of consumers in the market are differentiated by a willingness-to-pay-for-quality parameter α, uniformly distributed on $[0,1]$, such that a consumer with parameter α that purchases product i at price p_i has expected utility

$$\alpha u_i - p_i, \quad (2)$$

and will buy the product on the market with the maximum (2), if that is nonnegative, and otherwise will not buy a product. Under the standard condition $\Sigma_{i \in \mathcal{N}} q_i < 1$, the unique market equilibrium price per unit for each firm i's product is

$$p_i = u_i - \Sigma_{j \in \mathcal{N}} \min(u_i, u_j) \underline{q}_j; \quad (3)$$

(3) is derived in the appendix. Note that q_i is the share of the consumer market captured by firm i if it successfully brings its product to market; we will refer to parameter q_i as firm i's market share.

Hence firm i's expected profit (gross of fixed production costs) is

$$\pi_i = [u_i(1 - q_i) - \Sigma_{j \in \mathcal{N} \backslash i} \min(u_i, u_j) s_j(e_j, \mathbf{t}_j) q_j] \cdot s_i(e_i, \mathbf{t}_i) m q_i - c_i(e_i) - \Sigma_{j \in \mathcal{N} \backslash i} t_{ij}. \quad (4)$$

Each firm $i \in \mathcal{N}$ chooses its compliance $e_i \in [0,1]$ and testing of competitors $t_{ij} \geqslant 0$ for $j \in \mathcal{N} \backslash i$ to maximize (4), given its *beliefs* about competitors' compliance and testing decisions. (After exerting compliance effort e_i, firm i does not know with certainty whether its product is actually compliant (unless $e_i \in \{0,1\}$); instead the firm knows this likelihood e_i. The other firms $j \neq i$ cannot observe e_i. Whether or not a product is compliant (a credence attribute) can be observed only through testing. Hence although compliance effort occurs prior to testing, for purposes of analysis, this is a simultaneous-move game.) We focus on pure strategy Nash equilibria in compliance and testing by the firms.

As motivated by the review of related literature, we treat the regulator testing t_{Ri} of each firm $i \in \mathcal{N}$ as a parameter for sensitivity analysis, and assume that the firms correctly anticipate $\{t_{Ri}\}_{i \in \mathcal{N}}$.

Production quantity is not a decision variable in the base model analyzed in Section 3. That is equivalent to assuming that each firm produces at capacity, as in Chod and Rudi (2005), Anand and Girotra (2007), and Swinney et al. (2011), in which case mq_i may be interpreted as the capacity of firm i. That assumption is reasonable under a new product standard because firms' compliance efforts and the potential for blocking increase the expected selling price, which tends to strengthen a firm's incentive to produce at capacity.

Regarding consumers' utility from compliance with the product standard, the base model of consumer utility in (2) with fixed $\{u_i\}_{i \in \mathcal{N}}$ has two interpretations. First, consumers are indifferent to whether a product is compliant with the standard (which might be the case with a standard that benefits the environment and does not directly benefit a consumer). Second, consumers trust that products on the market comply with the standard, in which case u_i reflects the utility a consumer perceives in having a compliant product from firm i.

Section 4 extends the model and analysis, to allow consumers to value compliance and have rational expectations about the likelihood that a product in the

RIGHTSLINK

market is compliant, to allow a firm to choose its product quality apart from compliance, and to allow a firm to choose its production quantity, among other extensions.

3. Results

3.1. Testing by the Regulator

What is the impact of regulator testing on the firms' testing and compliance efforts? Proposition 1 shows that a small amount of regulator testing reduces firms' testing and has no impact on compliance; a large amount of regulator testing prevents firms' testing and can reduce compliance.

To state Proposition 1, consider an initial equilibrium in compliance and testing by the firms $\{\hat{e}_i, \hat{t}_{ji}\}_{i\in\mathcal{N},\, j\in\mathcal{N}\setminus i}$ when the regulator does not test ($t_{Ri} = 0$ for $i \in \mathcal{N}$) and define

$$\tau_i \equiv \max\{t_{Ri}\colon d_i(0,\ldots,0,t_{Ri}) \le d_i(\hat{t}_{1i},\ldots,\hat{t}_{Ni},0)\}, \quad \text{for } i \in \mathcal{N}. \tag{5}$$

Lemma 1 implies existence of that initial equilibrium $\{\hat{e}_i, \hat{t}_{ji}\}_{i\in\mathcal{N},\, j\in\mathcal{N}\setminus i}$.

Lemma 1. *For any given amount of regulator testing $\{t_{Ri}\}_{i\in\mathcal{N}}$, an equilibrium exists in the firms' testing and compliance efforts.*

The proofs of the results in this section are in the appendix.

Proposition 1(a) considers an increase in regulator testing from zero ($t_{Ri} = 0$ for $i \in \mathcal{N}$) to a "small" amount ($t_{Ri} \le \tau_i$ for $i \in \mathcal{N}$). "Small" is a relative term, meaning that the probability that the regulator would detect a noncompliant product if competitors did not test, $d_i(0,\ldots,0,t_{Ri})$ in (5), is smaller than the initial probability that the competitors detect a noncompliant product when the regulator does not test, $d_i(\hat{t}_{1i},\ldots,\hat{t}_{Ni},0)$ in (5). Insofar as the regulator is less effective than firms in testing, and insofar as firms are motivated to test their competitors, each τ_i is actually large. Similarly, Proposition 1(b) considers an increase in regulator testing from zero ($t_{Ri} = 0$ for $i \in \mathcal{N}$) to a "large" amount ($t_{Ri} > \bar{\tau}_i$ for $i \in \mathcal{N}$), where

$$\bar{\tau}_i \equiv \min\{t_{Ri}\colon (\partial/\partial t_{ji}) d_i(0,\ldots,0,t_{Ri}) \le 1/(m u_j), \quad \text{for } j \in \mathcal{N}\setminus i\}. \tag{6}$$

Proposition 1. (a) *A "small" amount of testing by the regulator ($t_{Ri} \le \tau_i$ for $i \in \mathcal{N}$) reduces the equilibrium testing by each firm while preserving each firm's equilibrium compliance and detection probability.*

(b) *A "large" amount of testing by the regulator ($t_{Ri} > \bar{\tau}_i$ for $i \in \mathcal{N}$) ensures that the firms do not test, $t_{ji} = 0$ for $i \in \mathcal{N}$, $j \in \mathcal{N}\setminus i$ and, for some parameters, strictly reduces the unique equilibrium compliance e_i of every firm $i \in \mathcal{N}$.*

The rationale for Proposition 1(a) is that testing by the regulator discourages firms from testing competitors' products, by reducing the marginal probability that a firm can "knock out" a competitor through its own testing effort. As the regulator increases testing of firm i from $t_{Ri} = 0$ to a "small" level $t_{Ri} \le \tau_i$, each competitor firm j optimally reduces its testing of firm i to the extent that the detection probability for firm i remains the same.[4] Every firm's optimal compliance effort and testing of competitors other than firm i depend on the detection probability for firm i, not how that detection probability is achieved through regulator versus competitors' testing of firm i. Hence for every firm, the initial (with zero regulator testing) equilibrium compliance effort and testing of competitors other than firm i remains optimal, so the detection probability for a noncompliant product for each firm also remains the same as in the initial equilibrium.

The rationale for Proposition 1(b) is similar in that at sufficiently large levels of regulator testing, the firms cease to test. To prove that regulator testing can strictly *decrease* compliance effort by all firms in the market, the proof of Proposition 1(b) provides a simple example with $N = 2$ firms, in which the regulator applies the same level of testing to both firms ($t_{R1} = t_{R2}$), and the firms are symmetric except that a violation by firm 2 is more difficult to detect, especially for the regulator. Without testing by the regulator ($t_{R1} = t_{R2} = 0$), each firm tests its competitor. When the regulator applies a large level of testing ($t_{R1} = t_{R2} > \max(\bar{\tau}_1, \bar{\tau}_2)$), the firms cease to test ($t_{12} = t_{21} = 0$) in the unique equilibrium. Because it is easier to detect a violation by firm 1, the regulator's common testing level increases the detection probability of firm 1 and decreases the detection probability for firm 2. The latter causes firm 2 to decrease its compliance effort. The decrease in firm 2's detection probability makes firm 2 more likely to bring its product to market, which reduces the expected market price, and thereby causes firm 1 to decrease its compliance effort. Firm 1 is no longer motivated to test firm 2's product because the regulator is doing so and because firm 1's detection probability is higher and its compliance lower, so firm 1 is less likely to bring its product to market, and hence less likely to benefit from knocking firm 2 out of the market.[5]

In short, Proposition 1 shows that a small amount of regulator testing will be ineffectual, and a large amount can be counterproductive. (Why would a regulator test at an ineffectual or counterproductive level? A small budget earmarked for testing might cause a regulator to choose an ineffectually small level of testing. Inability to commit to a testing level before firms choose their compliance efforts and utility from detecting violations might cause a regulator to choose a high testing level that reduces firms' compliance efforts.)

The policy implication of Proposition 1 is that relying on competitor testing (setting regulator testing $t_{Ri} = 0$

RIGHTSLINK

for $i \in \mathcal{N}$)[6] might be socially optimal. If firms test competitors when the regulator does not test, introducing a relatively "small" level of testing by the regulator reduces social welfare by increasing the total social cost of testing (crowding out the presumably more efficient testing by firms) without improving detection or compliance, according to Proposition 1(a).[7] To potentially increase detection or compliance, the regulator must incur an even higher testing cost, which is not worthwhile insofar as firms' detection probabilities and compliance efforts are large without regulator testing.

Therefore, in Section 3.2, we assume that the regulator does not test, and characterize conditions under which all firms' products are tested by competitors and the firms' compliance efforts are large. Under those conditions, regulator testing would crowd out testing by competitors and potentially reduce social welfare. When those conditions do not hold, regulator testing, targeted so as to complement competitor testing, can be useful, as explained at the end of Section 3.2.

3.2. Testing Only by Competitors

Proposition 2 highlights the critical importance of firms' quality levels $\{u_i\}_{i\in\mathcal{N}}$ in stimulating competitor testing and hence compliance. The quality parameter u_i can be interpreted as the utility that a consumer expects from firm i's product, which reflects the strength of firm i's brand, the consumer's utility from compliance with the standard (assuming the consumer trusts that a product on the market is compliant), and the consumer's utility from other quality attributes of the product.

Proposition 2. (a) *In any equilibrium, at least one firm tests a competitor ($t_{nk} > 0$ for some $n \in \mathcal{N}$ and $k \in \mathcal{N}\backslash n$) if and only if at least two firms have sufficiently high quality*

$$\min(u_i, u_j) > 1/\{q_j m q_i (\partial/\partial t_{ij}) d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}\} \quad \text{for some } j \in \mathcal{N} \text{ and } i \in \mathcal{N}\backslash j. \tag{7}$$

(b) *In any equilibrium, if firm j's quality is sufficiently low*

$$u_j \le 1/\Big\{q_j \max_{i\in\mathcal{N}\backslash j}[m q_i (\partial/\partial t_{ij}) d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}]\Big\}, \tag{8}$$

then firm j draws no testing and does not comply ($t_{ij} = 0$ and $e_j = 0$ for all $i \in \mathcal{N}\backslash j$).

The proof shows that firm i tests firm j's product (or vice versa) if *both* have sufficiently high quality (7) and no other party tests their products. The robust underlying phenomenon is that in a market with products of vertically differentiated quality, the boost in the price of product i from knocking out product j is proportional to the quality of the lower-quality product $\min(u_i, u_j)$.[8] Consequently, firm i is motivated to test firm j's product insofar as *both* firms' products are of high quality. As a further consequence, if firm j has sufficiently low quality (8), no competitor tests its product, so firm j has no incentive for compliance.

Under the additional assumption $c_i'(0) = 0$ for $i \in \mathcal{N}$, meaning that a firm can at low cost achieve some positive probability that its product will be compliant, (7) is the necessary and sufficient condition for at least one firm to exert some compliance effort. Under the additional assumption, any firm that draws testing exerts strictly positive compliance effort.

Proposition 2 shows that testing of competitors occurs in a large or concentrated market. Specifically, (7) tends to hold when the market size m is large or market share is concentrated in the hands of two firms (so $\max_{j\in\mathcal{N}, i\in\mathcal{N}\backslash j}\{q_j q_i\}$ is large). The rationale is that a firm j tends to draw testing when its market share q_j is large, so that knocking firm j out of the market substantially increases the market price for every other firm's product (3). Furthermore, a firm i is motivated to test competitors insofar as the market m or firm i's own market share q_i is large, because the increase in firm i's per-unit price p_i from knocking out a competitor is multiplied by firm i's production quantity mq_i.

Proposition 2 also shows that testing of competitors occurs when one firm has insider knowledge that makes a small amount of testing likely to be effective. Specifically, condition (7) holds and (8) is violated when $(\partial/\partial t_{ij}) d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}$ is sufficiently large. In practice, that could occur because firm i has insider knowledge (perhaps a tip-off from a supplier or other channel partner) regarding how competitor j's product is likely to be noncompliant, so that with minimal testing expenditure, firm i has a substantial probability of detecting a violation by competitor j.

As a corollary to Proposition 2(b), enforcement of a product standard through competitor testing strictly improves the profitability of firms with low quality (8). The counterfactual scenario has no product standard or, equivalently, a standard that is not enforced, so that firm i's expected profit is $(u_i - \Sigma_{j\in\mathcal{N}} \min(u_i, u_j) q_j) m q_i$.

Corollary. *A product standard, enforced through competitor testing, increases the expected profit of firms with low quality (8), which do not comply with the standard.*

The rationale is that a low-quality firm spends nothing on compliance, may choose not to test, and sells at a higher price when its higher-quality competitors knock each other out of the market.

Focusing on the case where the firms are symmetric, Proposition 3 characterizes the impact of the number of firms and their quality on the firms' equilibrium compliance and testing efforts, and the resulting detection probability for a noncompliant product. To ensure existence of a unique symmetric equilibrium, in which each firm exerts compliance effort $\hat{e}$ and tests each competitor at level $\hat{t}$, Proposition 3 assumes that the cost of compliance is strictly convex, high compliance

RIGHTSLINK

effort is costly and the detection probability function is component-wise sufficiently concave, for $e \in [0,1)$

$$c''(e) > 0 \quad \text{and} \quad \lim_{e \uparrow 1} c(e) > mu, \tag{9}$$

$$(\partial^2/\partial t_{ij}^2) d(\mathbf{t}_j) < -\max\{1/d(\mathbf{t}_j), mu/[(1-\bar{d})^3 c''(e)]\} \cdot N(\partial/\partial t_{ij})\, d(\mathbf{t}_j)^2. \tag{10}$$

Convexity in compliance cost and concavity in detection probability are natural in that one would expect a firm to prioritize the most cost effective activities. For example, to increase the likelihood of detecting noncompliance in a competitor product, a firm could: gather additional information about what failure modes are most likely (e.g., by identifying a competitor's riskiest suppliers); or increase the scope or sophistication of its tests. Similarly, to increase the likelihood that its product complies with the standard, a firm could: increase its care in product design, supplier selection or manufacturing; or subject its own product to more rigorous inspection. To the extent that activities differ in their cost effectiveness and that a firm prioritizes its activities accordingly, that convexity and concavity become more pronounced. In (10), greater convexity in the compliance cost lessens the degree of concavity required in the detection probability function.

Proposition 3. *Suppose the firms are symmetric and* (9)–(10) *hold. In the unique symmetric equilibrium, firms test* ($\hat{t} > 0$) *if and only if* $u > 1/[mq^2(\partial/\partial t_{ij}) d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}]$. *Compliance* $\hat{e}$ *increases with the firms' quality levels* u *and with the market size* m, *and decreases with the number of firms* N. *The detection probability for a noncompliant product increases with the number of firms* N *for* $N \le \bar{N}$ *and decreases with* N *for* $N \ge \bar{N}$.

The basic reason that equilibrium compliance effort $\hat{e}$ increases with the firms' quality levels u and with market concentration (a smaller number N of firms supplying the market) is that those exogenous factors drive up the market price (3). Hence each firm has greater value from bringing products to market, and correspondingly stronger incentive for compliance effort to increase the probability of doing so. Similarly, an increase in the market size m increases each firm's value from bringing a product to market, which stimulates compliance effort.

However, the proof of that monotonicity in equilibrium compliance effort $\hat{e}$ is subtle because, in contrast, equilibrium testing $\hat{t}$ and the resulting detection probability for a noncompliant product are not monotonic in the market size m, quality u, and number of firms N. Equilibrium testing $\hat{t}$ increases with the number of firms N for $N \le \bar{N}$ and decreases with N for $N \ge \bar{N}$ because reduced compliance strengthens a firm's incentive for testing when compliance is high (as is the case with few firms $N \le \bar{N}$) and weakens this incentive when compliance is low (as is the case with many firms $N \ge \bar{N}$). Reduced compliance has two countervailing effects on testing. First, a competitor's reduced compliance strengthens a firm's incentive for testing because testing is more likely to reveal noncompliance. This testing-incentive-strengthening effect is strong when the testing firm's compliance is high because then the firm is likely to bring products to market, so the firm's incentive for testing is sensitive to its competitor's reduced compliance. Second, a firm's reduced compliance weakens its incentive to test because the firm is more prone to being blocked from the market. This testing-incentive-weakening effect is weak when the competitor firm's compliance is high because then it is unlikely that testing will be effective in blocking the competitor firm's products from the market, so the incentive for testing is insensitive to the reduction to the testing firm's reduced compliance.

A synthesis of the propositions in Sections 3.1 and 3.2 reveals the conditions under which regulator testing can be useful, as opposed to socially inefficient. The latter occurs when firms' quality levels are high and the market size is large, because then—without regulator testing—all firms' products would be tested by competitors and their compliance efforts would be large (Proposition 3). Hence regulator testing would crowd out competitor testing (Proposition 1) and likely reduce social welfare as explained in the last two paragraphs of Section 3.1. When, to the contrary, the firms' quality levels are low or the market size is small, regulator testing can be useful. First, because firms do not test competitors' products (Proposition 2(a)), regulator testing is needed to detect and block noncompliant products from the market. Second, a regulator's sufficiently intense testing of a firm's product pushes the firm, that otherwise would not exert compliance effort, to do so. More generally, these benefits are captured by targeted regulator testing of the product of any firm with low quality or low market share (Proposition 2(b)). Such targeted regulator testing complements firm testing, which is directed to products of competitors with high quality and market share. If the social cost of noncompliance is sufficiently high, it may be useful for the regulator to test the products of a broader set of firms, including products that would otherwise be tested by competitors. For this to be socially beneficial, the regulator must test those products at a high level, as otherwise its testing will simply crowd out the more efficient testing by competitors (Proposition 1(a)).

4. Extensions

4.1. Endogenous Quality: Entrants

The corollary to Proposition 2(b) suggests that a product standard, enforced through competitor testing

RIGHTSLINK

(alone), will cause entry by low-quality firms with little incentive for compliance. Proposition 4 confirms that such entry occurs, in the following, extended version of our model. Suppose that a firm can enter the market with production quantity mq. An entrant chooses whether to develop a product with low quality u^l or high quality u^h, where $u^l < u^h$,

$$u^l \le 1 \bigg/ \left[mq^2 \max_{j \in \mathcal{N} \setminus i} (\partial / \partial t_{ji})\, d_i(\mathbf{t}_i) \Big|_{\mathbf{t}_i = \mathbf{0}} \right], \tag{11}$$

and the entry cost is k^l or k^h depending on whether the entrant chooses low or high quality, respectively, where $k^l < k^h$. A firm enters if doing so yields nonnegative expected profit. Initially (prior to adoption of the product standard), the market is in an equilibrium with more than two high-quality firms; a sufficient condition for existence of such an equilibrium is that their entry cost $k^h \le \min(u^h(1-3q)mq, k^l u^h/u^l)$. Similarly, assume that $k^l < u^l(1-2q)mq$ so a low-quality firm might enter. Finally, assume that testing costs are sufficiently low that, after adoption of the product standard, at least one incumbent firm does some testing.

Proposition 4. *A product standard, enforced through competitor testing, causes entry by at least one firm with low quality u^l (the type of firm that does not comply) and no firms with high quality u^h if detection is sufficiently easy*

$$(\partial/\partial t_{ij}) d_j(\mathbf{t}_j) \ge \gamma, \quad \text{if } d_j(\mathbf{t}_j) \le \underline{d}, \tag{12}$$

and compliance is sufficiently costly

$$c_i(e_i) \ge \underline{c}, \quad \text{for } e_i \ge \underline{e},$$

for $i \in \mathcal{N}$, where $\gamma \in (0, \infty)$, $\underline{d} < 1$, $\underline{c} \in (0, \infty)$ and $\underline{e} \in (0, 1)$.

The proofs of all the results in this section are in the online supplement.

4.2. Endogenous Quality: Incumbents

We now turn to the impact of a product standard, enforced through competitor testing, on the quality chosen by incumbent firms. Suppose that each firm $i \in \mathcal{N}$ privately chooses a quality level $u_i \in \{u^1, u^2, .., u^M\}$ simultaneously with compliance e_i. The cost to firm i of choosing quality u^n is k_i^n. If a firm is indifferent between two quality levels, the firm chooses the higher-quality level.

Our main insight is that enforcement of a product standard through competitor testing reduces quality. Proposition 5 establishes that the quality reduction occurs for all firms, when firms are symmetric. As in the above corollary and Proposition 4, the counterfactual scenario has no product standard or, equivalently, a standard that is not enforced.

Proposition 5. *If the firms are symmetric, a product standard, enforced through competitor testing, reduces quality in any symmetric equilibrium.*

As is clear from the proof, a parallel results holds with asymmetric firms: a product standard, enforced through competitor testing, reduces the quality of firms with highest equilibrium quality in the counterfactual scenario.

In reality, firms may observe competitors' investments in quality (or advertising expenditures, or other costly efforts to strengthen a brand) before deciding how much to test competitors' products. Consider, therefore, a sequential-move version of the game, with observable investment in quality followed by testing. A new product standard, enforced through competitor testing, can strictly reduce the quality investment of every incumbent firm, and do so to greater extent than in the initial version of the game with unobserved quality; a numerical example is given in the online supplement. Quality investment tends to be lower in the sequential-move game with observable quality than in the initial version of the game because lowering one's quality reduces the incentive for testing by competitors. The caveat is that lower quality may signal lower compliance effort, and thereby, indirectly, increase the incentive for testing by competitors.

4.3. Consumer Utility from Compliance

Suppose that a consumer who purchases product i at price p_i has expected utility (2) in the event that the product is noncompliant, and has expected utility

$$\alpha u_i (1 + \Delta) - p_i \tag{13}$$

in the event that the product is compliant, where $\Delta \ge 0$. We refer to Δ as a consumer's utility from compliance. Recall that consumers have heterogeneous levels of the parameter α, which may be attributed to their heterogeneous income levels; a consumer with high income has low price sensitivity, corresponding to high α.

Further suppose that consumers form expectations regarding firm i's compliance effort $\tilde{e}_i$, the testing applied to firm i by its competitors and the regulator $\tilde{\mathbf{t}}_i$, and the resulting probability $\tilde{e}_i / s_i(\tilde{e}_i, \tilde{\mathbf{t}}_i)$ that firm i's product is compliant given that it is available for sale in the market. Hence, the expected utility of a consumer with valuation parameter α from purchasing firm i's product is $\alpha u_i [1 + \Delta \tilde{e}_i / s_i(\tilde{e}_i, \tilde{\mathbf{t}}_i)] - p_i$. We focus on rational expectations equilibria that are consistent, that is, satisfy $\tilde{e}_i = e_i$ and $\tilde{\mathbf{t}}_i = \mathbf{t}_i$ for all $i \in N$. Therefore the market equilibrium price for product i and firm i's expected profit are given by (3) and (4), respectively, with u_n replaced by

$$\tilde{u}_n \equiv u_n [1 + \Delta \tilde{e}_n / s_n(\tilde{e}_n, \tilde{\mathbf{t}}_n)], \quad \text{for } n \in \{i, j\}. \tag{14}$$

All of the preceding Propositions hold, with explicit incorporation of the utility from compliance. Proposition 1 holds with $u_j(1+\Delta)$ replacing u_j in (6). Proposition 2(a) holds under the conservative (for purposes of evaluating the effectiveness of competitor testing) assumption that if multiple equilibria exist, including

RIGHTSLINK

one with zero testing, then zero testing occurs. Even without that assumption, (7) is sufficient for at least one firm to test a competitor, and (7) with $\min(u_i, u_j)\cdot(1+\Delta)$ replacing $\min(u_i, u_j)$ is sufficient for no firm to test a competitor. Proposition 2(b) holds with $u_j(1+\Delta)$ replacing u_j in (8). Proposition 3 holds with the following generalized conditions to ensure a unique equilibrium: In (9), $mu\Delta/(1-\bar{d})^3$ replaces 0; in (10), $u(1+\Delta)^2$ replaces u, and $(1-\bar{d})^3c''(e) - mu\Delta$ replaces $(1-\bar{d})^3\cdot c''(e)$. Furthermore, Proposition 3 extends in that compliance $\hat{e}$ increases with a consumer's utility from compliance Δ. Proposition 4 holds. Proposition 5 holds with an upper bound on Δ.

In sum, relying on competitor testing tends to be effective when consumers value compliance (Δ is large) in that competitor testing stimulates high compliance effort (Proposition 3). Conversely, when consumers value neither compliance nor the product itself (Δ and u_i for $i \in \mathcal{N}$ are small), relying on competitor testing is ineffective because firms choose not to test competitors' products (Proposition 2).

We adopt this generalized formulation in all subsequent sections.

4.4. Other Penalties for Noncompliance

In the event that a firm's product is shown to be noncompliant, instead of blocking the sale of the product, a regulator could simply notify consumers that the product is noncompliant. Notification could be accomplished by publishing a list of noncompliant products or preventing a firm from labeling its product as compliant. For example, the U.S. Department of Energy employs this "labeling" approach in enforcement of the voluntary Energy Star standard. In this labeling scenario, firm i always sells mq_i units, but at a reduced price in the event that its product is shown to be noncompliant. The unique market equilibrium price per unit for each firm i's product is

$$p_i = \underline{u}_i - \Sigma_{j\in\mathcal{N}} \min(\underline{u}_i, \underline{u}_j)q_j,$$

where $\underline{u}_j$ is the random variable

$$\underline{u}_j \equiv \begin{cases} \tilde{u}_j & \text{with probability } s_j(e_j, \mathbf{t}_j), \\ u_j & \text{with probability } 1 - s_j(e_j, \mathbf{t}_j), \end{cases}$$

with $\tilde{u}_j$ defined in (14). For a given vector of the firms' compliance and testing efforts, we assume that $\underline{u}_i$ and $\underline{u}_j$ for $j \neq i$ are independent. Therefore, firm i's expected profit (gross of fixed production costs) is

$$\begin{aligned}\pi_i = \big[\tilde{u}_i(1-q_i) - \Sigma_{j\in\mathcal{N}\setminus i}\{s_j(e_j,\mathbf{t}_j)\min(\tilde{u}_i,\tilde{u}_j) \\ + [1-s_j(e_j,\mathbf{t}_j)]\min(\tilde{u}_i,u_j)\}q_j\big]s_i(e_i,\mathbf{t}_i) \\ \times mq_i + \big[u_i(1-q_i) - \Sigma_{j\in\mathcal{N}\setminus i}\{s_j(e_j,\mathbf{t}_j)\min(u_i,\tilde{u}_j) \\ + [1-s_j(e_j,\mathbf{t}_j)]\min(u_i,u_j)\}q_j\big] \\ \times [1-s_i(e_i,\mathbf{t}_i)]mq_i - c_i(e_i) - \Sigma_{j\in\mathcal{N}\setminus i}t_{ij}. \qquad (15)\end{aligned}$$

Another motivation for this labeling scenario is that, in the absence of a government-imposed standard, firms may test competitors' products in order to detect and inform consumers of a defect or false claim. For example, competitor testing alone serves to enforce voluntary product labeling standards, devised by industry and other nongovernmental organizations, for "natural" and "organic" personal care products (Story 2008, Struck 2008).

Suppose that when a firm's product is shown to be noncompliant, the firm incurs a fine $f \geq 0$. That introduces the term $-f[1-s_i(e_i,\mathbf{t}_i)]$ into the objective function for firm i in (4) for the scenario with blocking and in (15) for the scenario with labeling. That "fine" f may also incorporate costs associated with civil lawsuits, criminal penalties, long-term reputational damage, or (with blocking) the cost to safely destroy and dispose of a product shown to be noncompliant.

The propositions in Section 3 hold with the fine $f \geq 0$ and blocking. Propositions 1–3 hold. Proposition 3 extends in that the symmetric equilibrium compliance increases with the fine f, and there exists $\bar{f}$ such that the symmetric equilibrium detection probability increases with f for $f \leq \bar{f}$ and decreases with f for $f \geq \bar{f}$.

With the penalty of the fine $f \geq 0$ and labeling, the propositions in Section 3 largely hold. Proposition 1(a) and the first part of Proposition 1(b) hold. Proposition 2(b) holds when $\min_{n\in\mathcal{N}}\{u_n\}/(1+\Delta)$ is added to the right hand side of (8). The comparative statics in Proposition 3, including those for the fine, hold for the symmetric equilibrium with the largest compliance level; nonuniqueness arises because of existence of an equilibrium with zero compliance and testing. Existence of that equilibrium nullifies Proposition 2(a).

Should a regulator strip offending products of labels (such as "Energy Star"), instead of blocking them from the market? Doing so reduces compliance effort by incumbent firms, in the context of Proposition 3. Specifically, for any fine $f \geq 0$, any symmetric equilibrium compliance under labeling is lower than the unique symmetric equilibrium compliance under blocking. However, with asymmetric firms, the extended Proposition 2(b) implies that the switch to labeling might cause firms with low—but not too low—quality to draw testing by a competitor and exert some compliance effort. Labeling reverses the results in the corollary and Proposition 4: A product standard, enforced through competitor testing and labeling, *decreases* the expected profit of firms that do not comply and, in the setting of Proposition 4, does *not* cause entry by low-quality, noncompliant firms. The rationale for that reversal is that, in the labeling scenario, a firm that draws testing from competitors and therefore chooses to exert compliance effort earns higher profit; consumers become willing to pay a higher price for its

RIGHTSLINK

product than if the firm did not draw testing. To summarize, the switch to labeling eliminates the problem of entry by low-quality, noncompliant firms, but may reduce incumbents' compliance efforts.

Another argument for blocking is that in a labeling scenario, with a different sequence of decisions than assumed in this paper, a firm may choose not to report a competitor's violation. In Li and Peeters (2017), an incumbent firm decides whether to incur cost to learn the quality level of an entrant's product and whether to report that information to consumers. Afterward, the incumbent and entrant set their prices, then produce to meet consumer demand. The main result in Li and Peeters (2017) is that, in a specified parameter region, the incumbent chooses not to report a defect in the entrant's product, because doing so would cause the entrant to set a lower price and thereby reduce the incumbent's sales. In contrast, in this paper, firms' production quantities are set in advance, so reporting a competitor's violation to consumers is always optimal.

4.5. Endogenous Production Quantities

Suppose that each firm $i \in \mathcal{N}$ chooses its quantity mq_i, unobserved by its competitors, at cost $C_i(q_i)$, where $C_i(\cdot)$ is a strictly increasing function. That is equivalent to having each firm $i \in \mathcal{N}$ choose its target market share q_i, because the market size m is an exogenous parameter. The lead time for production is sufficiently long that a firm chooses its quantity prior to observing which products are blocked. For purposes of analysis, this is captured by assuming that firms make quantity, in addition to compliance and testing, decisions in a simultaneous-move game. The propositions in Section 3 are robust in this extension. Proposition 1 holds. Proposition 2 holds where: in the sufficient condition for a firm to draw testing (7), q_n for $n \in \{j, i\}$ is firm n's equilibrium production quantity when no firm draws testing; the sufficient condition for no firm to draw testing is (7) being violated where $(1+\Delta)m$ replaces $q_j m q_i$; and (8) is replaced by $u_j \le 1/\{(1+\Delta)m \max_{i\in\mathcal{N}\setminus j}[(\partial/\partial t_{ij})d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}]\}$. The online supplement provides numerical results consistent with Proposition 3.

4.6. Fixed Costs for Testing

Suppose that each firm $i \in \mathcal{N}$ must incur a fixed cost $\phi_i \ge 0$ in order to test the product of any competitor-firm $j \in \mathcal{N}\setminus i$ at a strictly positive level $t_{ij} > 0$. This captures the setting in which firm i needs to acquire equipment and/or technical expertise to test its competitors' products; ϕ_i represents the fixed cost associated with obtaining this competency. The base model formulation in Section 2, wherein $\phi_i = 0$, represents the case where the firm already has this competency, for example, due to prior investments in its own product development process (Wiel and McMahon 2005). Alternatively, $\phi_i = 0$ may arise because firm i relies on an outside laboratory to conduct tests. Similarly, suppose the regulator incurs a fixed cost $\phi_R \ge 0$ if $t_{Rj} > 0$ for any $j \in \mathcal{N}$.

The propositions in Section 3 largely hold. Proposition 1(a) holds, with a more complex expression for τ_i reflecting that a marginal increase in regulator testing can motivate a discontinuous drop in a firm's testing from a strictly positive level to zero. Proposition 1(b) holds. In fact, a stronger version of Proposition 1(b) holds in that, for some parameters, introducing testing by the regulator strictly reduces the detection probability for a noncompliant product for every firm, in addition to strictly reducing compliance effort for every firm. Regarding Proposition 2(a), (7) is sufficient for at least one firm to test a competitor provided that the fixed cost of testing is not too large, and (7) with $\min(u_i, u_j)(1+\Delta)$ replacing $\min(u_i, u_j)$ is sufficient for no firm to test a competitor. Proposition 2(b) holds with $u_j(1+\Delta)$ replacing u_j in (8). Proposition 3 holds provided that the fixed cost of testing is not too large.

4.7. Alternative Detection Probability Function

In the base model formulated in Section 2, the detection probability function is component-wise strictly increasing. That requires that when multiple firms test the product of firm i, their testing activities are not perfectly duplicative. That is natural if the firms have different sources of information regarding what failure modes are likely or different testing capabilities. The base model also captures a scenario wherein firms conduct the same noisy test that randomly fails to detect a violation, so independent repetition of the test increases the probability that at least one detects a violation. In another scenario, firms pay a testing service provider to test the product of a competitor, the testing service provider prioritizes the most cost-effective testing activities, and therefore the detection probability is a strictly increasing and concave function of firms' total expenditure $\Sigma_{i\in\mathcal{N}\setminus j} t_{ij}$; that no-duplication scenario also is captured in the base model.

Let us now consider the case of perfect duplication in testing activities that is not captured in the base model. Suppose that the detection probability for a noncompliant product $d_i(\mathbf{t}_i) = D_i(\max_{n\in R\cup\mathcal{N}\setminus i}\{t_{ni}\})$, where $D_i(\cdot)$ is strictly increasing, strictly concave, differentiable and satisfies $D_i(0) = 0$ and $\lim_{t\to\infty} D_i(t) = \bar{d}$, where $\bar{d} \in (0,1)$. Furthermore, in (10), $D(t)$ replaces $d(\mathbf{t}_j)$, $D'(t)$ replaces $(\partial/\partial t_{ij})d(\mathbf{t}_j)$, and $D''(t)$ replaces $(\partial^2/\partial t_{ij}^2)d(\mathbf{t}_j)$. That alternative detection function is not componentwise strictly increasing, is not continuously differentiable, and does not satisfy (1).

Nevertheless, the preceding Propositions hold. The caveat is that Proposition 1(a) is modified in that the expression for τ_i is more complex (reflecting that a marginal increase in regulator testing of firm i can motivate a discontinuous drop in competitor testing

RIGHTSLINK

from a high level to zero, and thus strictly reduce the detection probability for firm i) and that "maintains the equilibrium testing" replaces "reduces the equilibrium testing." The central message of Proposition 1(a), that a small of amount of testing by the regulator may be socially inefficient, continues to hold, but because such regulator testing duplicates rather than crowds out testing by the firms. Propositions 1(b), 2–5 hold. The modified Propositions described in the preceding extensions subsections also hold.

5. Concluding Remarks

Insights from the stylized model in this paper suggest that testing by government regulatory authorities to detect violations of a product standard may be detrimental to social welfare—when that testing is directed toward products that would otherwise be tested by competitors. Testing by a regulator crowds out testing by competitors and thereby increases the overall social cost of testing, because regulator testing is less efficient, as documented in Section 1. With a small budget—as is common in practice (Bruschia 2008, Smith 2008)—regulator testing fails to improve firms' compliance efforts and fails to increase the detection probability for a noncompliant product. With a large budget, regulator testing can strictly reduce firms' compliance efforts and strictly reduce the detection probability for a noncompliant product.

Competitor testing alone is effective in enforcing a product standard if consumers highly value the product or compliance with the product standard, or if the market is concentrated or large. Market integration and harmonization of product standards (increasing the size of the market governed by a product standard) spurs firms to test their competitors and exert greater compliance effort, as does each of the other listed factors.

When relying on competitor testing to detect violations, stronger penalties may be detrimental. Increasing the fine increases compliance effort, but can reduce competitor testing and the detection probability for a noncompliant product. Blocking the sale of a product shown to be noncompliant—rather than simply publicizing that the product is noncompliant—increases compliance effort by incumbent firms, but may cause entry by low-quality, noncompliant firms that do not draw testing from competitors.

A caveat is that the stylized model in this paper does not account for collusion or reputational concerns that, in reality, deter some firms from reporting competitors' violations. For example, German automakers secretly agreed to commonly adopt technology inadequate to clean diesel exhaust to emissions standards. None of the automakers reported a competitor's violation, because doing so would reveal its own violation. The collusion came to light after a nongovernmental organization's (NGO) testing detected violations (Dohmen and Hawranek 2017, Ewing 2017). Similarly, firms might fail to report competitors' violations because they have a common reputation (meaning that a firm would be harmed by publicity of a competitor's violation, as in some food industries (Winfree and McCluskey 2005)) or a common supplier (so a firm might be harmed if a competitor's violation was traced to the common supplier (Lawrence et al. 2017)).

Another caveat is that with a voluntary standard like Energy Star, relying on competitor testing is effective only if consumers have favorable beliefs. In a pernicious equilibrium, consumers think that products are noncompliant, firms have no incentive to test competitors' products and—without testing—have no incentive to comply with the standard. Indeed, the United States relied on competitor testing for enforcement of Energy Star and consumers trusted the Energy Star label—until a General Accountability Office report and related publicity of violations damaged that trust. In response, the U.S. Department of Energy instituted a new requirement for firms to pay for third-party testing of their products to use the Energy Star label (Gaffigan 2007, GAO 2010, Rosner 2010).

Third-party testing has qualitatively the same impacts on firms' compliance and testing efforts as does regulator testing. Specifically, in the model in this paper, one may interpret the parameter t_{Ri} as the level of third-party (e.g., independent testing laboratory or NGO) testing for firm i's product. Proposition 1 holds with "third-party" substituted for "regulator" and for any specified levels of $\{t_{Ri}\}_{i\in\mathcal{N}}$. Of course, the specified testing levels $\{t_{Ri}\}_{i\in\mathcal{N}}$ and detection probability functions $\{d_i()\}_{i\in\mathcal{N}}$ should reflect the objectives, capabilities, and budget constraints of the third party, which differ from those of a regulator. For example, a fiscally constrained government may impose higher t_{Ri} with third-party testing than with regulator testing, by requiring firm i to pay for the third-party testing—if firm i can afford to do so. To avoid putting small producers out of business, the United States exempts them from some third-party product safety testing requirements (Nord 2010). The results in this paper suggest that such small producers do not draw testing by competitors, so regulator or third-party testing is needed to incentivize them to comply with credence standards.

Acknowledgments

This research was motivated by conversations with Andre Algazi and Karl Palmer from the California Department of Toxic Substances Control, Jeff Hunts from the California Integrated Waste Management Board, Walter Alcorn and Jason Linnell from the National Center for Electronics Recycling, and Chris Smith, secretary of the European RoHS enforcement network. Carl Shapiro and Ben Hermalin provided useful comments on an earlier version.

RIGHTSLINK

Appendix

Derivation of Market Equilibrium Prices (3). Index the firms such that quality u_i increases with i. In any equilibrium, prices p_i increase with i, consumers with low willingness-to-pay-for-quality $\alpha \in [0, 1-\Sigma_{j=1}^N \underline{q}_j)$ do not purchase, and the marginal consumer with $\alpha = 1-\Sigma_{j=1}^N \underline{q}_j$ purchases the product with lowest price and quality, indexed $l = \arg\min\{u_j\colon \underline{q}_j > 0\}$, which implies $p_l = u_l(1-\Sigma_{j=1}^N \underline{q}_j)$, establishing (3) for product $i = l$. For every other product in the market, $i > l$ with $\underline{q}_i > 0$, the marginal buyer has $\alpha = 1-\Sigma_{j=i}^N \underline{q}_j$ and is indifferent between product i and the one with next-highest price and quality, indexed $k = \arg\max\{u_j\colon \underline{q}_j > 0, j < i\}$:

$$u_i(1-\Sigma_{j=i}^N \underline{q}_j) - p_i = u_k(1-\Sigma_{j=i}^N \underline{q}_j) - p_k. \tag{A.1}$$

Having established (3) for product l, proceed by induction, assuming (3) holds for product k:

$$p_k = u_k - \Sigma_{j\in\mathcal{N}} \min(u_k, u_j)\underline{q}_j. \tag{A.2}$$

Together, (A.1) and (A.2) imply (3) for product i.

Proof of Lemma 1. A pure strategy Nash equilibrium exists in a game in which every player has a compact, convex strategy set and a utility function that is continuous in all players' strategies and quasiconcave in his own strategy (theorem 8.D.3 in Mas-Colell et al. 1995). The assumption that $c_i(\cdot)$ is convex for $i \in \mathcal{N}$ ensures that it is continuous and, with the assumption that $d_i(\mathbf{t}_i)$ is continuously differentiable for $i \in \mathcal{N}$, ensures that firm i's objective function (4) is continuous in all the firms' compliance and testing strategies $\{e_k, t_{kj}\}_{k\in\mathcal{N},\, j\in\mathcal{N}\setminus k}$. To establish that firm i's objective function (4) is quasiconcave in firm i's compliance and testing strategy $\{e_i, t_{ij}\}_{j\in\mathcal{N}\setminus i}$, observe that the assumption that $d_i(\mathbf{t}_i)$ is componentwise strictly increasing, continuously differentiable and satisfies (1) implies

$$(\partial^2/\partial t_{ni}\partial t_{ki}) d_i(\mathbf{t}_i) < 0, \quad \text{for } n, k \in \mathcal{N}\setminus i, \tag{A.3}$$

and, in particular, $d_j(\mathbf{t}_j)$ is a strictly concave function of t_{ij} for $i \in \mathcal{N}$ and $j \in \mathcal{N}\setminus i$. A sum of concave functions is concave. A concave function remains concave when multiplied by a nonnegative constant. A convex function ($c_i(\cdot)$) becomes concave when multiplied by a negative constant. Therefore, it remains to show that $e_i d_j(\mathbf{t}_j)$ is a concave function of $\{e_i, t_{ij}\}$ for each $j \in \mathcal{N}\setminus i$. The Hessian matrix of $e_i d_j(\mathbf{t}_j)$,

$$\begin{bmatrix} e_i(\partial^2/\partial t_{ij}^2) d_j(\mathbf{t}_j) & (\partial/\partial t_{ij}) d_j(\mathbf{t}_j) \\ (\partial/\partial t_{ij}) d_j(\mathbf{t}_j) & 0 \end{bmatrix},$$

is negative semidefinite due to (A.3), $d_j(\mathbf{t}_j)$ being componentwise strictly increasing, and the nonnegativity of e_i, so $e_i d_j(\mathbf{t}_j)$ is indeed a concave function of $\{e_i, t_{ij}\}$. To see that each firm $i \in \mathcal{N}$ has a compact, convex strategy set, recall that firm $i \in \mathcal{N}$ is constrained to choose $e_i \in [0,1]$ and $t_{ij} \geq 0$ for $j \in \mathcal{N}\setminus i$. Firm $i \in \mathcal{N}$ is also, effectively, constrained to choose $t_{ij} \leq u_i$ for $j \in \mathcal{N}\setminus i$; by inspection of (4), firm i would have a strictly negative objective value if firm i set $t_{ij} > u_i$ for any $j \in \mathcal{N}\setminus i$, whereas firm i is guaranteed a nonnegative objective value by setting $e_i = 0$ and $t_{ij} = 0$ for all $j \in \mathcal{N}\setminus i$. Therefore, by the aforementioned theorem, a pure strategy Nash equilibrium exists in the firms' testing and compliance efforts $\{e_k, t_{kj}\}_{k\in\mathcal{N},\, j\in\mathcal{N}\setminus k}$. □

Proof of Proposition 1. (a) Each firm j's objective π_j is quasiconcave in its compliance and testing $\{e_j, t_{ji}\}_{i\in\mathcal{N}\setminus j}$ as shown in the proof of Lemma 1. Hence $\{e_j, t_{ji}\}_{j\in\mathcal{N},\, i\in\mathcal{N}\setminus j}$ is an equilibrium if and only if the first-order conditions for compliance and testing hold for each firm $j \in \mathcal{N}$:

$$\frac{\partial \pi_j}{\partial e_j} = [u_j(1-q_j) - \Sigma_{i\in\mathcal{N}\setminus j} \min(u_j, u_i) s_i(e_i, \mathbf{t}_i) q_i] d_j(\mathbf{t}_j) m q_j$$
$$- c_j'(e_j) \begin{cases} \leq 0 & \text{if } e_j = 0, \\ = 0 & \text{if } e_j \in (0,1), \\ \geq 0 & \text{if } e_j = 1; \end{cases} \tag{A.4}$$

$$\frac{\partial \pi_j}{\partial t_{ji}} = \min(u_j, u_i) s(e_j, \mathbf{t}_j) q_i m q_j (1-e_i) \frac{\partial d_i(\mathbf{t}_i)}{\partial t_{ji}}$$
$$-1 \begin{cases} \leq 0 & \text{if } t_{ji} = 0, \\ = 0 & \text{if } t_{ji} > 0, \end{cases} \quad \text{for } i \in \mathcal{N}\setminus j. \tag{A.5}$$

Recall that $\{\hat{e}_j, \hat{t}_{ji}\}_{j\in\mathcal{N},\, i\in\mathcal{N}\setminus j}$ denotes the initial equilibrium in compliance and testing by the firms when the regulator does not test, $t_{Ri} = 0$ for $i \in \mathcal{N}$. Consider the introduction of regulator testing $\mathbf{t}_R = \langle t_{R1}, t_{R2}, .., t_{RN}\rangle$ that satisfies $t_{Ri} \leq \tau_i$ for $i \in \mathcal{N}$. We will construct best response testing levels for the firms $\{\hat{t}'_{ji}\}_{i\in\mathcal{N},\, j\in\mathcal{N}\setminus i}$ that are reduced from the initial testing levels so as to preserve the detection probability for a noncompliant product:

$$\hat{t}'_{ji} \leq \hat{t}_{ji}, \quad \text{for } j \in \mathcal{N},\ i \in \mathcal{N}\setminus j, \tag{A.6}$$

$$\hat{t}'_{ji} < \hat{t}_{ji}, \quad \text{for } i \in \mathcal{N} \text{ with } t_{Ri} > 0, \text{ for some } j \in \mathcal{N}\setminus i, \tag{A.7}$$

$$d_i(\hat{\mathbf{t}}'_i, t_{Ri}) = d_i(\hat{\mathbf{t}}_i, 0), \quad \text{for } i \in \mathcal{N}, \tag{A.8}$$

with vector notation $\hat{\mathbf{t}}_i = \langle \hat{t}_{1i}, \hat{t}_{2i}, .., \hat{t}_{i-1,i}, \hat{t}_{i+1,i}, .., \hat{t}_{Ni}\rangle$ for the initial equilibrium testing of firm i by other firms and $\hat{\mathbf{t}}'_i = \langle \hat{t}'_{1i}, \hat{t}'_{2i}, .., \hat{t}'_{i-1,i}, \hat{t}'_{i+1,i}, .., \hat{t}'_{Ni}\rangle$ for their best response testing of firm i after the introduction of regulator testing. We will then prove that $\{\hat{e}_j, \hat{t}'_{ji}\}_{j\in\mathcal{N},\, i\in\mathcal{N}\setminus j}$ is an equilibrium, that is, each firm's initial compliance level is preserved in equilibrium. To do so, it will be sufficient to prove that

$$(\partial/\partial t_{ji}) d_i(\hat{\mathbf{t}}'_i, t_{Ri}) \leq (\partial/\partial t_{ji}) d_i(\hat{\mathbf{t}}_i, 0) \text{ and that inequality}$$
$$\text{holds with equality if } \hat{t}'_{ji} > 0, \text{ for } j \in \mathcal{N},\ i \in \mathcal{N}\setminus j. \tag{A.9}$$

Together, (A.8), (A.9), and the fact that (A.4)–(A.5) hold at the initial equilibrium $\{\hat{e}_j, \hat{t}_{ji}\}_{j\in\mathcal{N},\, i\in\mathcal{N}\setminus j}$ with zero regulator testing imply that (A.4)–(A.5) hold for $\{\hat{e}_j, \hat{t}'_{ji}\}_{j\in\mathcal{N},\, i\in\mathcal{N}\setminus j}$ with regulator testing $\mathbf{t}_R$.

The construction of τ_i in (5) implies that $d_i(\mathbf{0}, t_{Ri}) \leq d_i(\hat{\mathbf{t}}_i, 0)$ for $i \in \mathcal{N}$. If $t_{Ri} = 0$, then set $\hat{\mathbf{t}}'_i = \hat{\mathbf{t}}_i$. If $t_{Ri} > 0$, then $0 < d_i(\mathbf{0}, t_{Ri}) \leq d_i(\hat{\mathbf{t}}_i, 0)$, so at least one competitor tested firm i in the initial equilibrium. Let N_i denote the number of firms that tested firm i in the initial equilibrium. Suppose that the firms are indexed so that firms $1, .., N_i$ are doing that testing, that is, $\hat{t}_{ji} > 0$ for $j = 1, .., N_i$ and $\hat{t}_{ji} = 0$ for $j > N_i$. Let $\mathcal{T}_1(t, \mathbf{t}_i)$ denote the vector of competitor testing levels for firm i with first component t and second through Nth components identical to those of $\mathbf{t}_i$. In other words, $\mathcal{T}_1(t, \mathbf{t}_i)$ transforms a vector $\mathbf{t}_i$ by substituting t for its first component. Set

$$\hat{t}'_{1i} = \min\{t \geq 0\colon d_i(\mathcal{T}_1(t, \hat{\mathbf{t}}_i), t_{Ri}) \geq d_i(\hat{\mathbf{t}}_i, 0)\}.$$

As d_i is continuous and componentwise strictly increasing, one of the following two cases must occur. In the

RIGHTSLINK

first case, $d_i(\mathcal{T}_1(\hat{t}'_{1i},\hat{\mathbf{t}}_i),t_{Ri}) = d_i(\hat{\mathbf{t}}_i,0)$. In the second case, $d_i(\mathcal{T}_1(\hat{t}'_{1i},\hat{\mathbf{t}}_i),t_{Ri}) > d_i(\hat{\mathbf{t}}_i,0)$ and $\hat{t}'_{1i}=0$. In the first case, set $\hat{\mathbf{t}}'_i = \mathcal{T}_1(\hat{t}'_{1i},\hat{\mathbf{t}}_i)$. In the second case, sequentially reduce the testing of firm i by firms $j=2,3,\ldots$ in the same manner, until the detection probability for a noncompliant product by firm i falls to the initial level $d_i(\hat{\mathbf{t}}_i,0)$. Specifically, for $j\in\mathcal{N}\backslash i$, let $\mathcal{T}_j(t,\mathbf{t}_i)$ denote the vector of competitor testing levels for firm i with components 1 through $j-1$ set to zero, component j set to t, and all other components identical to those of $\mathbf{t}_i$. In other words, $\mathcal{T}_j(t,\mathbf{t}_i)$ transforms a vector $\mathbf{t}_i$ by substituting t for its jth component, and 0 for components 1 through $j-1$. Iteratively, starting with $j=2$, set

$$\hat{t}'_{ji} = \min\{t\ge 0\colon d_i(\mathcal{T}_j(t,\hat{\mathbf{t}}_i),t_{Ri})\ge d_i(\hat{\mathbf{t}}_i,0)\}.$$

If $d_i(\mathcal{T}_j(\hat{t}'_{ji},\hat{\mathbf{t}}_i),t_{Ri}) = d_i(\hat{\mathbf{t}}_i,0)$, then stop with $\hat{\mathbf{t}}'_i=\mathcal{T}_j(\hat{t}'_{ji},\hat{\mathbf{t}}_i)$. Otherwise, $d_i(\mathcal{T}_j(\hat{t}'_{ji},\hat{\mathbf{t}}_i),t_{Ri}) > d_i(\hat{\mathbf{t}}_i,0)$ and $\hat{t}'_{ji}=0$. Increment $j=j+1$, until $d_i(\mathcal{T}_j(\hat{t}'_{ji},\hat{\mathbf{t}}_i),t_{Ri}) = d_i(\hat{\mathbf{t}}_i,0)$ and then stop with $\hat{\mathbf{t}}'_i=\mathcal{T}_j(\hat{t}'_{ji},\hat{\mathbf{t}}_i)$. Observe that $d_i(\mathcal{T}_j(\hat{t}'_{ji},\hat{\mathbf{t}}_i),t_{Ri}) = d_i(\hat{\mathbf{t}}_i,0)$ is achieved for $j\le N_i$ because d_i is continuous and componentwise strictly increasing, and $d_i(\mathbf{0},t_{Ri})\le d_i(\hat{\mathbf{t}}_i,0)$. Follow this process to construct $\hat{\mathbf{t}}'_i$ for each $i\in\mathcal{N}$ with the properties (A.6)–(A.8).

It remains to prove (A.9). The proof is by contradiction, and uses the assumptions that d_i is continuously differentiable, component-wise strictly increasing and satisfies (1). Suppose $(\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}'_i,t_{Ri}) > (\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}_i,0)$. Then, because $(\partial/\partial t_{ji})d_i$ is continuous with respect to t_{ji} and d_i is component-wise strictly increasing, there exists some $\varepsilon>0$ such that, with $\hat{\mathbf{t}}''_i$ defined to have $\hat{t}''_{ji}=\hat{t}'_{ji}+\varepsilon$ and $\hat{t}''_{ki}=\hat{t}'_{ki}$ for $k\in\mathcal{N}\backslash\{i,j\}$, $(\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}''_i,t_{Ri}) > (\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}_i,0)$ and $d_i(\hat{\mathbf{t}}''_i,t_{Ri}) > d_i(\hat{\mathbf{t}}'_i,t_{Ri})$. The latter, with (1) and (A.8), implies that $(\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}''_i,t_{Ri}) < (\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}_i,0)$, a contradiction. Similarly, for the case that $\hat{t}'_{ji}>0$, suppose $(\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}'_i,t_{Ri}) < (\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}_i,0)$. Then, because $(\partial/\partial t_{ji})d_i$ is continuous with respect to t_{ji} and d_i is component-wise strictly increasing, there exists some $\varepsilon>0$ such that, with $\hat{\mathbf{t}}''_i$ defined to have $\hat{t}''_{ji}=\hat{t}'_{ji}-\varepsilon$ and $\hat{t}''_{ki}=\hat{t}'_{ki}$ for $k\in\mathcal{N}\backslash\{i,j\}$, $(\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}''_i,t_{Ri}) < (\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}_i,0)$ and $d_i(\hat{\mathbf{t}}''_i,t_{Ri}) < d_i(\hat{\mathbf{t}}'_i,t_{Ri})$. The latter, with (1) and (A.8), implies that $(\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}''_i,t_{Ri}) > (\partial/\partial t_{ji})d_i(\hat{\mathbf{t}}_i,0)$, a contradiction.

(b) First, we show that if $t_{Ri}>\bar{\tau}_i$ for $i\in\mathcal{N}$, then any equilibrium has $t_{ji}=0$ for $i\in\mathcal{N}$ and $j\in\mathcal{N}\backslash i$. Each $\bar{\tau}_i$ is a finite, nonnegative constant, due to our assumptions that $d_i(\mathbf{t}_i)$ is componentwise strictly increasing, continuously differentiable and satisfies (1) and $\lim_{t_{ji}\to\infty} d_i(\mathbf{t}_i)=\bar{d}$ for $j\in\{R,\mathcal{N}\backslash i\}$, which imply that $\lim_{t_{Ri}\to\infty}(\partial/\partial t_{ji})d_i(0,..,0,t_{Ri})=0$. Those assumptions also imply that for any $\mathbf{t}_i$ with regulator testing $t_{Ri}>\bar{\tau}_i$,

$$mu_j(\partial/\partial t_{ji})d_i(\mathbf{t}_i)-1 \le mu_j(\partial/\partial t_{ji})d_i(0,..,0,t_{Ri})-1<0, \quad \text{for every } j\in\mathcal{N}\backslash i. \tag{A.10}$$

Also observe that

$$\begin{aligned}(\partial/\partial t_{ji})\pi_j &= \min(u_i,u_j)s_j(e_j,\mathbf{t}_j)q_i m q_j(1-e_i)(\partial/\partial t_{ji})d_i(\mathbf{t}_i)-1\\ &\le mu_j(\partial/\partial t_{ji})d_i(\mathbf{t}_i)-1.\end{aligned} \tag{A.11}$$

Therefore, $t_{Ri}>\bar{\tau}_i$ implies that $(\partial/\partial t_{ji})\pi_j<0$, so in any equilibrium $t_{ji}=0$. Second, we provide an example with $N=2$ firms and regulator testing $t_{Ri}>\bar{\tau}_i$ for $i\in\{1,2\}$ in which every firm's unique equilibrium compliance e_i is strictly lower than with zero regulator testing. Let $m=u_1=u_2=1$, $q_1=q_2=0.48$, $\bar{d}=0.99$, $c_i(e)=0.15e^2$ for $i\in\{1,2\}$,

$$d_1(t_{21},t_{R1})=\min\Big(\sqrt{8(t_{21}+t_{R1})},2\bar{d}\Big(\sqrt{[300(t_{21}+t_{R1})]^2+300(t_{21}+t_{R1})}-300(t_{21}+t_{R1})\Big)\Big)$$

and

$$d_2(t_{12},t_{R2})=\min\Big(\sqrt{2t_{12}+t_{R2}/150},2\bar{d}\Big(\sqrt{[300(t_{21}+t_{R1})]^2+300(t_{21}+t_{R1})}-300(t_{21}+t_{R1})\Big)\Big).$$

Then, under regulator testing $(t_{R1},t_{R2})=(0.12,0.12)$, the unique equilibrium has zero testing by the firms and compliance $(e_1,e_2)=(0.0835,0.0213)$. Under $(t_{R1},t_{R2})=(0,0)$, the unique equilibrium has testing by the firms $(t_{21},t_{12})=(0.07850,0.00188)$ and compliance $(e_1,e_2)=(0.0866,0.0380)$. □

Proof of Proposition 2. The proof has five steps. First, we establish properties of an equilibrium. Second, we establish properties of the detection probability function $d_j(\mathbf{t}_j)$. Third, we establish that (7) implies that in any equilibrium either $t_{nj}>0$ for some $n\in\mathcal{N}\backslash j$ or $t_{ki}>0$ for some $k\in\mathcal{N}\backslash i$. Fourth, we establish that if in equilibrium $t_{nk}>0$ for some $k\in\mathcal{N}$ and $n\in\mathcal{N}\backslash k$, then (7) holds. Fifth, we establish that (8) implies that firm j draws no testing. Thus, steps 1–4 address part (a) and step 5 addresses part (b). First, any equilibrium $\{e_j,t_{ji}\}_{j\in\mathcal{N},\,i\in\mathcal{N}\backslash j}$ must satisfy the first-order necessary conditions for compliance by firm $j\in\mathcal{N}$ and testing by firm $i\in\mathcal{N}\backslash j$:

$$\frac{\partial\pi_j}{\partial e_j} = [u_j(1-q_j)-\Sigma_{l\in\mathcal{N}\backslash j}\min(u_i,u_l)s_l(e_l,\mathbf{t}_l)q_l]d_j(\mathbf{t}_j)mq_j - c'_j(e_j)\begin{cases}\le 0 & \text{if } e_j=0,\\ =0 & \text{if } e_j\in(0,1),\\ \ge 0 & \text{if } e_j=1;\end{cases} \tag{A.12}$$

$$\frac{\partial\pi_i}{\partial t_{ij}} = \min(u_i,u_j)s(e_i,\mathbf{t}_i)q_j m q_i(1-e_j)\frac{\partial d_j(\mathbf{t}_j)}{\partial t_{ij}} - 1\begin{cases}\le 0 & \text{if } t_{ij}=0,\\ =0 & \text{if } t_{ij}>0.\end{cases} \tag{A.13}$$

If firm j draws no testing ($t_{ij}=0$ for $i\in\mathcal{N}\backslash j$), then $d_j(\mathbf{t}_j)=0$ and consequently, in equilibrium $e_j=0$. Second, observe that our assumption that $d_j(\mathbf{t}_j)$ is componentwise strictly increasing, continuously differentiable, and satisfies (1) implies that $(\partial^2/\partial t_{nj}\partial t_{kj})d_j(\mathbf{t}_j)<0$ for $n\in\mathcal{N}\backslash j$ and $k\in\mathcal{N}\backslash j$. This implies $(\partial/\partial t_{ij})d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}} > (\partial/\partial t_{ij})d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\hat{\mathbf{t}}_j}$ where $\hat{t}_{nj}>0$ for some $n\in\mathcal{N}\backslash j$. Third, consider the case in which (7) holds. Consider a $j\in\mathcal{N}$ and $i\in\mathcal{N}\backslash j$ such that the inequality in (7) holds. Suppose in equilibrium $t_{nj}=t_{ki}=0$ for all $n\in\mathcal{N}\backslash j$ and $k\in\mathcal{N}\backslash i$. This implies that in equilibrium $e_j=e_i=0$ (from step 1). Let $\{\hat{e}_i,\hat{\mathbf{t}}_i\}_{i\in\mathcal{N}}$ denote this equilibrium. Then

$$\begin{aligned}&(\partial/\partial t_{ij})\pi_i|_{(e_n,\mathbf{t}_n)=(\hat{e}_n,\hat{\mathbf{t}}_n)\text{ for } n\in\mathcal{N}}\\ &\quad=\min(u_i,u_j)q_j m q_i(\partial/\partial t_{ij})d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}-1>0,\end{aligned} \tag{A.14}$$

where the equality follows from the expression for $(\partial/\partial t_{ij})\pi_i$ in (A.13), and the inequality in (A.14) follows from the inequality in (7). Because (A.14) contradicts (A.13), we conclude that in any equilibrium $t_{nj}>0$ for some $n\in\mathcal{N}\backslash j$ and/or $t_{ki}>0$ for some $k\in\mathcal{N}\backslash i$. Thus, if $t_{nj}=0$ and for all $n\in\mathcal{N}\backslash\{i,j\}$

RIGHTSLINK

and $t_{ki}=0$ and for all $k\in\mathcal{N}\backslash\{i,j\}$, then $t_{ij}>0$ and/or $t_{ji}>0$; that is, firm i tests firm j's product (or vice versa) if no other party tests firm i or j. Fourth, consider the case in which in equilibrium $t_{nk}>0$ for some $k\in\mathcal{N}$ and $n\in\mathcal{N}\backslash k$. Let $\{\hat{e}_i,\hat{\mathbf{t}}_i\}_{i\in\mathcal{N}}$ denote this equilibrium. Suppose (7) does not hold. For $j\in\mathcal{N}$ and $i\in\mathcal{N}\backslash j$ such that $t_{ij}>0$,

$$\begin{aligned}&(\partial/\partial t_{ij})\pi_i|_{(e_n,\mathbf{t}_n)=(\hat{e}_n,\hat{\mathbf{t}}_n)\text{ for }n\in\mathcal{N}}\\&\quad=\min(u_i,u_j)s_i(\hat{e}_i,\hat{\mathbf{t}}_i)q_jmq_i(1-\hat{e}_j)(\partial/\partial t_{ij})d_j(\hat{\mathbf{t}}_j)-1, \quad (A.15)\\&\quad<\min(u_i,u_j)q_jmq_i(\partial/\partial t_{ij})d_j(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}-1\le 0, \quad (A.16)\end{aligned}$$

where the equality follows from the expression for $(\partial/\partial t_{ij})\pi_i$ in (A.13), the first inequality follows from step 2, and the second inequality follows from (7) being violated. Because (A.16) contradicts that (A.13) holds with equality, we conclude that (7) holds. Fifth, consider the case in which (8) holds. Suppose in equilibrium that firm j draws testing $t_{ij}>0$ for some $i\in\mathcal{N}\backslash j$. Let $\{\hat{e}_i,\hat{\mathbf{t}}_i\}_{i\in\mathcal{N}}$ denote the equilibrium. Then (A.16) holds for the same reasons in step 4, with the exception that the last inequality holds because (8) holds. We conclude that firm j does not draw testing ($t_{ij}=0$ for all $i\in\mathcal{N}\backslash j$), and so does not comply $e_j=0$ (from step 1). □

Lemma 2 is useful in the proof of Proposition 3. Let $s^0(e,t)$ denote $s(e,\mathbf{t})$, $d^0(t)=d(\mathbf{t})$, $d^1(t)=(\partial/\partial t_{ij})d(\mathbf{t})$, and $d^2(t)=(\partial^2/\partial t_{ij}^2)d(\mathbf{t})$ for $i\in\mathcal{N}$ and $j\in\mathcal{N}\backslash i$, where $\mathbf{t}=\langle t,t,..t,0\rangle$ denotes the vector wherein each firm exerts testing effort t and the regulator does not test. Let

$$\begin{aligned}f_1(e,t)&=mus^0(e,t)q^2(1-e)d^1(t)-1,\\f_2(e,t)&=mu(1-[1+(N-1)s^0(e,t)]q)qd^0(t)-c'(e).\end{aligned}$$

$f_1(e,t)$ is the first derivative of firm i's profit function with respect to t_{ij} and $f_2(e,t)$ is the first derivative with respect to e_i, when each firm $j\in\mathcal{N}$ chooses compliance $e_j=e$ and testing $t_{jk}=t$ for $k\in\mathcal{N}\backslash j$. Define for $t>0$,

$$\begin{aligned}\underline{\mathbf{e}}_0(t)&=(2d^0(t)-1-\sqrt{1-4d^0(t)/[mud^1(t)q^2]})/[2d^0(t)],\\\bar{\mathbf{e}}_0(t)&=(2d^0(t)-1+\sqrt{1-4d^0(t)/[mud^1(t)q^2]})/[2d^0(t)],\end{aligned}$$

and let $\underline{\mathbf{e}}_0(0)=\lim_{t\downarrow 0}\underline{\mathbf{e}}_0(t)$ and $\bar{\mathbf{e}}_0(0)=\lim_{t\downarrow 0}\bar{\mathbf{e}}_0(t)$. Let $\bar{t}$ denote the unique solution to $d^0(t)/d^1(t)=muq^2/4$. If $t>\bar{t}$, then no value of e satisfies $f_1(e,t)=0$; otherwise, $f_1(e,t)=0$ has two roots in e: $\underline{\mathbf{e}}_0(t)$ and $\bar{\mathbf{e}}_0(t)$. Note that $f_2(e,t)$ is strictly increasing in t and strictly decreasing in e with $\lim_{e\uparrow\bar{e}}f_2(e,t)<0$. Let $\underline{t}=\inf_{t\ge 0}\{t\colon f_2(0,t)>0\}$. If $t>\underline{t}$, then let $\breve{e}_0(t)$ denote the unique solution to $f_2(e,t)=0$,and note that $\breve{e}_0(t)>0$; otherwise, let $\breve{e}_0(t)=0$. Let $\underline{\mathbf{e}}(d)=\underline{\mathbf{e}}_0(t)|_{t\text{ such that }d^0(t)=d}$, $\bar{\mathbf{e}}(d)=\bar{\mathbf{e}}_0(t)|_{t\text{ such that }d^0(t)=d}$, $\breve{e}(d)=\breve{e}_0(t)|_{t\text{ such that }d^0(t)=d}$, $\bar{\bar{d}}=d^0(\bar{t})$ and $\underline{d}=d^0(\underline{t})$. Our assumption that $d(\cdot)$ is continuously differentiable and component-wise strictly increasing implies existence of $\underline{\mathbf{e}}(d)$ and $\bar{\mathbf{e}}(d)$ for $d\in[0,\bar{\bar{d}}]$. Note that in the symmetric case, inequality (7) simplifies to $u>1/[mq^2(\partial/\partial t_{ij})d(\mathbf{t}_j)|_{\mathbf{t}_j=\mathbf{0}}]$.

Lemma 2. *Suppose the firms are symmetric and* (9)–(10) *hold. If* (7) *is violated, then the unique symmetric equilibrium in compliance and detection probability* $(\hat{e},\hat{d})=(0,0)$. *Otherwise, the unique symmetric equilibrium* $(\hat{e},\hat{d})$ *has* $\hat{d}\in(0,\bar{\bar{d}}]$ *and one of the following*:

$$\hat{e}=\underline{\mathbf{e}}(\hat{d})=\breve{e}(\hat{d}), \quad (A.17)$$
$$\hat{e}=\bar{\mathbf{e}}(\hat{d})=\breve{e}(\hat{d}). \quad (A.18)$$

Furthermore, $\underline{\mathbf{e}}(\cdot)$ *is continuous and strictly increasing and* $\bar{\mathbf{e}}(\cdot)$ *is continuous and strictly decreasing;* $\breve{e}(d)$ *is continuous on* $d\in[0,\bar{\bar{d}})$ *and increasing in* d*, strictly so on* $d\in(\underline{d},\bar{\bar{d}})$*. Finally,* $\breve{e}(0)=0$ *and* $\underline{\mathbf{e}}(0)<0<\bar{\mathbf{e}}(0)$.

Proof of Lemma 2. The proof proceeds in seven steps. First, we establish necessary conditions for a symmetric equilibrium. Second, we show that these conditions are sufficient. Third, we establish properties of the functions $\breve{e}(d)$, $\underline{\mathbf{e}}(d)$ and $\bar{\mathbf{e}}(d)$. Fourth, we show that $f_1(\breve{e}_0(t),t)$ is strictly decreasing in t. Fifth, we show that if (7) is violated, then the unique symmetric equilibrium has zero compliance and testing. Sixth, we show that if (7) is satisfied, then a symmetric equilibrium must satisfy either (A.17) or (A.18). Seventh, we show that if (7) is satisfied, then a unique symmetric equilibrium exists. First, we establish necessary conditions for a symmetric equilibrium. In a symmetric equilibrium (e,t), each firm $j\in\mathcal{N}$ chooses compliance $e_j=e$ and testing $t_{jk}=t$ for $j\in\mathcal{N}$ and $k\in\mathcal{N}\backslash j$. If firm i anticipates that the remaining firms $j\in\mathcal{N}\backslash i$ will choose compliance $e_j=e$ and testing $t_{jk}=t$ for $j\in\mathcal{N}$ and $k\in\mathcal{N}\backslash j$, then for compliance $e_i=e$ and testing $t_{ij}=t$ for $j\in\mathcal{N}$ to be a best response for firm i, the following first-order conditions must be satisfied

$$(\partial/\partial t_{ij})\pi_i|_{e_n=\tilde{e}_n=e,\,t_{nl}=\tilde{t}_{nl}=t\text{ for }n\in\mathcal{N}\text{ and }l\in\mathcal{N}\backslash n}=f_1(e,t)\le 0, \quad (A.19)$$
$$(\partial/\partial e_i)\pi_i|_{e_n=\tilde{e}_n=e,\,t_{nl}=\tilde{t}_{nl}=t\text{ for }n\in\mathcal{N}\text{ and }l\in\mathcal{N}\backslash n}=f_2(e,t)\le 0, \quad (A.20)$$

where (A.19) must hold with equality if $t>0$ and (A.20) must hold with equality if $e>0$. Second, we establish that any solution to (A.19)–(A.20) is a symmetric equilibrium. If firm i anticipates that the remaining firms $j\in\mathcal{N}\backslash i$ will choose compliance $e_j=e$ and testing $t_{jk}=t$ for $k\in\mathcal{N}\backslash j$, then any solution to the first-order conditions for firm i must for $j\in\mathcal{N}\backslash i$ have $t_{ij}=\tau$ for some $\tau\ge 0$. We can write firm i's expected profit under compliance e_i and testing level τ as

$$\begin{aligned}\pi_i(e_i,\tau)&=u\{1-[1+(N-1)s_o(e,t,\tau)]q\}\\&\quad\cdot mqs^0(e_i,t)-c(e_i)-(N-1)\tau, \quad (A.21)\end{aligned}$$

where $s_o(e,t_a,t_b)=1-d_o(t_a,t_b)(1-e)$, where $d_o(t_a,t_b)$ denotes the detection probability when all firms but one chooses testing level t_a, one firm chooses testing level t_b and the regulator does not test, $d_o(t_a,t_b)=d(t_a,..t_a,t_b,t_a,..t_a,0)$. The assumptions that $c(\cdot)$ is strictly convex and $(\partial^2/\partial t_{ij}^2)d(\mathbf{t}_j)<-(N-1)mu(\partial/\partial t_{ij})d(\mathbf{t}_j)^2/[(1-\bar{d})^2c''(e)]$ imply that π_i is jointly strictly concave in (e_i,τ). Therefore, if (A.19)–(A.20) are satisfied, then firm i's best response is $(e_i,\tau)=(e,t)$. Third, we establish properties of the functions $\breve{e}(d)$, $\underline{\mathbf{e}}(d)$ and $\bar{\mathbf{e}}(d)$. Because $(\partial^2/\partial t_{ij}^2)d(\mathbf{t}_j)<-N(\partial/\partial t_{ij})d(\mathbf{t}_j)^2/d^0(t)$, $\underline{\mathbf{e}}_0(t)$ is strictly increasing and $\bar{\mathbf{e}}_0(t)$ is strictly decreasing in t. Therefore, because $d(\cdot)$ is component-wise strictly increasing and continuously differentiable, $\underline{\mathbf{e}}(d)$ is continuous and strictly increasing in d and $\bar{\mathbf{e}}(d)$ is continuous and strictly decreasing in d. Because $f_2(\cdot,\cdot)$ is continuous, $\breve{e}_0(t)$ is continuous on $t\in[0,\infty)$. By the implicit function theorem, $\breve{e}_0(t)$ is strictly increasing in t for $t\in(\underline{t},\infty)$

$$\begin{aligned}&(\partial/\partial t)\breve{e}_0(t)\\&\quad=\{mu[(N-1)q^2s^0(e,t)^2d^0(t)^2]+s^0(e,t)^2c''(e)\}^{-1}\\&\qquad\times mu(N-1)qs^0(e,t)^2[1-Nq+2(N-1)(1-e)d^0(t)q]d^1(t)\\&\quad>0. \quad (A.22)\end{aligned}$$

RIGHTSLINK

Therefore, because $d(\cdot)$ is component-wise strictly increasing and continuously differentiable, $\breve{e}(d)$ is continuous and strictly increasing in $d \in (\underline{d}, \bar{d})$. Let $f_n(e,0)$ denote $\lim_{t\downarrow 0} f_n(e,t)$ for $n=1,2$. Fourth, we establish that $f_1(\breve{e}_0(t),t)$ is strictly decreasing in t. Note that

$$\begin{aligned}(\partial/\partial t) f_1(\breve{e}_0(t),t) \\ = mu\{-[1-\breve{e}_0(t)]^2 d^1(t)^2 - \{(1-2d^0(t)[1-\breve{e}_0(t)]) - \Delta[1-2\breve{e}_0(t)]\} \\ \cdot d^1(t)\breve{e}_0'(t) + [s^0(\breve{e}_0(t),t) + \Delta\breve{e}_0(t)][1-\breve{e}_0(t)]d^2(t)\}q^2. \quad \text{(A.23)}\end{aligned}$$

Using (A.22) and (A.23), with some effort it is possible to show that $(\partial^2/\partial t_{ij}^2)d(\mathbf{t}_j) < -2mu \times (\partial/\partial t_{ij})d(\mathbf{t}_j)^2/[(1-\bar{d})^3 c''(e)]$ for $e \in [0,1)$ implies that $(\partial/\partial t) f_1(\breve{e}_0(t),t) < 0$. Fifth, suppose (7) is violated. Then $f_1(0,0) \le 0$ and $f_2(0,0) \le 0$, so $(\hat{e},\hat{t}) = (0,0)$ is an equilibrium. By Proposition 2, no equilibrium exists with $\hat{t} > 0$, so $(\hat{e},\hat{t}) = (0,0)$ is the unique equilibrium. Sixth, suppose (7) holds. Because $f_1(0,0) > 0$, $(e,t) = (0,0)$ is not an equilibrium. Because $f_2(e,0) < 0$ for $e \in (0,1]$, an equilibrium cannot have $t = 0$. Thus, in any equilibrium (A.19) must hold with equality. Thus, a symmetric equilibrium must satisfy $\hat{t} \in (0,\bar{t}]$, and $\hat{e} = \underline{\mathbf{e}}_0(\hat{t}) = \breve{e}_0(\hat{t})$ or $\hat{e} = \bar{\mathbf{e}}_0(\hat{t}) = \breve{e}_0(\hat{t})$. Therefore, a symmetric equilibrium must satisfy $\hat{d} \in (0,\bar{\bar{d}}]$, and (A.17) or (A.18). Seventh, suppose (7) holds. From step 6, this implies that a symmetric equilibrium (e,t) must have $t > 0$. From the analysis in step 1, a symmetric equilibrium must have $(e,t) = (\breve{e}_0(t),t)$ where t satisfies

$$f_1(\breve{e}_0(t),t) = 0, \quad \text{(A.24)}$$

and, from step 2, any such solution is an equilibrium. To establish that there exists an unique symmetric equilibrium it is sufficient to show that there exists a unique solution to (A.24). Assumption (7) implies $f_1(\breve{e}_0(0),0) > 0$; furthermore, $\lim_{t\uparrow\infty} f_1(\breve{e}_0(t),t) < 0$. Therefore, the existence of a unique solution to (A.24) follows from the fact that $f_1(\breve{e}_0(t),t)$ is strictly decreasing in t (as shown in step 4). Finally, it is straightforward to verify that (7) implies $\underline{\mathbf{e}}_0(0) < 0 < \bar{\mathbf{e}}_0(0)$, which in turn implies $\underline{\mathbf{e}}(0) < 0 < \bar{\mathbf{e}}(0)$. □

Proof of Proposition 3. From Lemma 2, the unique symmetric equilibrium detection probability $\hat{d} > 0$ (equivalently, $\hat{t} > 0$) if and only if (7) holds. Furthermore, if (7) is violated, then the unique symmetric equilibrium compliance $\hat{e} = 0$. Therefore, in the remainder, we consider the case where (7) holds. First, we demonstrate the comparative statics for the number of firms N. By the implicit function theorem, $\breve{e}(d)$ is decreasing in N. Let $\tilde{N} = \max_{N\in\{2,3,..\}}\{N\colon \breve{e}(\bar{\bar{d}}) \ge 1 - 1/(2\bar{\bar{d}})\}$. With some abuse of notation, let $(\hat{e}(N), \hat{d}(N))$ denote the unique symmetric equilibrium. If $N \le \tilde{N}$, then $\hat{d}$ is the unique solution to $\bar{\mathbf{e}}(d) - \breve{e}(d) = 0$. Furthermore,

$$\bar{\mathbf{e}}(d) - \breve{e}(d) \ge 0, \quad \text{if and only if } d \in [0,\hat{d}]. \quad \text{(A.25)}$$

For any $N_0 < N_1 \le \tilde{N}$, $0 = [\bar{\mathbf{e}}(\hat{d}(N_0)) - \breve{e}(\hat{d}(N_0))]|_{N=N_0} \le [\bar{\mathbf{e}}(\hat{d}(N_0)) - \breve{e}(\hat{d}(N_0))]|_{N=N_1}$, where the inequality follows because $\breve{e}(d)$ is decreasing in N. This implies that

$$\hat{d}(N_0) \le \hat{d}(N_1) \quad \text{(A.26)}$$

(from (A.25)). Thus,

$$\hat{e}(N_0) = \bar{\mathbf{e}}(\hat{d}(N_0)) \ge \bar{\mathbf{e}}(\hat{d}(N_1)) = \hat{e}(N_1), \quad \text{(A.27)}$$

where the inequality follows from (A.26) and the fact that $\bar{\mathbf{e}}(\cdot)$ is decreasing. By similar argument, for any $\tilde{N} < N_2 < N_3$,

$$\hat{d}(N_2) \ge \hat{d}(N_3), \quad \text{(A.28)}$$

$$\hat{e}(N_2) \ge \hat{e}(N_3). \quad \text{(A.29)}$$

Furthermore, for any $N_1 \le \tilde{N} < N_2$,

$$\hat{e}(N_1) \ge 1 - 1/(2\bar{\bar{d}}) \ge \hat{e}(N_2). \quad \text{(A.30)}$$

Together, (A.27), (A.29), and (A.30) imply that $\hat{e}$ is decreasing in N. Together, (A.26) and (A.28) imply that $\hat{d}$ is increasing in N for $N \le \bar{N}$ and decreasing in N for $N \ge \bar{N}$, where $\bar{N} = \arg\max_{N\in\{2,3,..\}}\{\hat{d}\}$. Second, we demonstrate the comparative statics for the quality level u. By the implicit function theorem, $\breve{e}(d)$ is increasing in u. With some effort, one can show that $\bar{\mathbf{e}}(d)$ is increasing in u and that $\underline{\mathbf{e}}(d)$ is decreasing in u for $d \in [0,\bar{\bar{d}}]$; recall that $\hat{d} \le \bar{\bar{d}}$ (by Lemma 2). Because $\underline{\mathbf{e}}(\cdot)$ and $\breve{e}(\cdot)$ are continuous, $\underline{\mathbf{e}}(0) < \breve{e}(0)$, $\underline{\mathbf{e}}(d) < \bar{\mathbf{e}}(d)$, and $\hat{d}$ satisfies either $\underline{\mathbf{e}}(\hat{d}) = \breve{e}(\hat{d})$ or $\bar{\mathbf{e}}(\hat{d}) = \breve{e}(\hat{d})$,

$$\underline{\mathbf{e}}(d) < \breve{e}(d), \quad \text{for } d \in [0,\hat{d}). \quad \text{(A.31)}$$

With some abuse of notation, let $(\hat{e}(u), \hat{d}(u))$ denote the unique symmetric equilibrium under quality level u. From (A.31), for any $u_h > u_l$ and any $d < \hat{d}(u_l)$

$$\underline{\mathbf{e}}(d)|_{u=u_h} \le \underline{\mathbf{e}}(d)|_{u=u_l} < \breve{e}(d)|_{u=u_l} \le \breve{e}(d)|_{u=u_h}. \quad \text{(A.32)}$$

If $\hat{e}(u_h) = \underline{\mathbf{e}}(\hat{d}(u_h))|_{u=u_h}$, then $\underline{\mathbf{e}}(\hat{d}(u_h))|_{u=u_h} = \breve{e}(\hat{d}(u_h))|_{u=u_h}$ and (A.32) together imply that $\hat{d}(u_h) \ge \hat{d}(u_l)$. Therefore, $\hat{e}(u_h) = \breve{e}(\hat{d}(u_h))|_{u=u_h} \ge \breve{e}(\hat{d}(u_l))|_{u=u_h} \ge \breve{e}(\hat{d}(u_l))|_{u=u_l} = \hat{e}(u_l)$. It remains to show that if $\hat{e}(u_h) = \bar{\mathbf{e}}(\hat{d}(u_h))|_{u=u_h}$, then $\hat{e}(u_h) \ge \hat{e}(u_l)$. Suppose $\hat{e}(u_h) = \bar{\mathbf{e}}(\hat{d}(u_h))|_{u=u_h}$ and $\hat{e}(u_l) = \bar{\mathbf{e}}(\hat{d}(u_l))|_{u=u_l}$. Let $D(u_a,u_b)$ denote the unique solution to

$$\bar{\mathbf{e}}(D)|_{u=u_a} - \breve{e}(D)|_{u=u_b} = 0. \quad \text{(A.33)}$$

Note that when $u_a = u_b = u_n$ for $n \in \{h,l\}$, there is only one solution to (A.33) and $\hat{d}(u_n) = D(u_n,u_n)$. We will show that

$$\hat{d}(u_l) \ge D(u_l,u_h). \quad \text{(A.34)}$$

The proof is by contradiction. Suppose that $\hat{d}(u_l) < D(u_l,u_h)$. Then

$$\bar{\mathbf{e}}(D(u_l,u_h))|_{u=u_l} < \breve{e}(D(u_l,u_h))|_{u=u_l} \le \breve{e}(D(u_l,u_h))|_{u=u_h}, \quad \text{(A.35)}$$

where the first inequality holds because the continuity of $\underline{\mathbf{e}}(\cdot)$ and $\breve{e}(\cdot)$, $\bar{\mathbf{e}}(0) > \breve{e}(0)$, and the uniqueness of $(\hat{e},\hat{d})$ imply $\bar{\mathbf{e}}(d) < \breve{e}(d)$ for $d > \hat{d}$; the second inequality holds because $\breve{e}(d)$ is decreasing in u. Because (A.35) contradicts the definition of $D(u_l,u_h)$, we have established (A.34). By similar argument,

$$\hat{d}(u_h) \ge D(u_l,u_h). \quad \text{(A.36)}$$

We conclude that $\hat{e}(u_l) = \bar{\mathbf{e}}(\hat{d}(u_l))|_{u=u_l} \le \bar{\mathbf{e}}(D(u_l,u_h))|_{u=u_l} = \breve{e}(D(u_l,u_h))|_{u=u_h} \le \breve{e}(\hat{d}(u_h))|_{u=u_h} = \hat{e}(u_h)$, where the first inequality follows from (A.34) and $\bar{\mathbf{e}}(\cdot)$ being decreasing; the second inequality follows from (A.36) and $\breve{e}(\cdot)$ being increasing. By similar argument, if $\hat{e}(u_h) = \bar{\mathbf{e}}(\hat{d}(u_h))|_{u=u_h}$ and $\hat{e}(u_l) = \underline{\mathbf{e}}(\hat{d}(u_l))|_{u=u_l}$, then $\hat{e}(u_l) \le \hat{e}(u_h)$. Third, by argument parallel to that for the comparative statics for u, $\hat{e}$ is increasing in the market size m. □

RIGHTSLINK

Endnotes

[1] Regulatory authorities in the United States and European Union encourage competitor testing by offering anonymity for firms that report competitors' violations of restrictions on hazardous substances (Bruschia 2008, Smith 2008). Although "it's not unusual for competitors to turn each other in" to the U.S. Consumer Product Safety Commission (CPSC) for violating product safety standards (Overfelt 2006), anonymous reports from competitors are explicitly excluded from the public database of reports about safety violations in consumer products (see http://www.saferproducts.gov/faq-business.aspx, last accessed July 1, 2018).

[2] The European Union's Registration, Evaluation, Authorisation and Restriction of Chemicals (REACH) regulation has expanded the number of restricted substances—from 6 under RoHS to 62 under REACH as of October 2016. In addition, REACH requires that *all* chemicals in a product be clearly identified and registered. Thus, REACH is multiplying the number of possible failure modes for a product. Whereas RoHS applies to electronics, REACH applies to all products.

[3] Whirlpool identified such a failure mode in refrigerator-freezers of its competitor LG, motivating the U.S. Department of Energy to prevent LG from selling those refrigerator-freezers under the Energy Star label (Vestel 2009, GAO 2010, Brown 2012).

[4] The preservation of the detection probability relies on the assumptions that the detection function is differentiable, componentwise strictly increasing, and satisfies (1), which are used only in the proof of Proposition 1(a). Section 4.7 relaxes those assumptions and extends Proposition 1(a).

[5] The regulator's being less effective in testing than the firms is not essential. The online supplement provides an example wherein testing by a *more* effective regulator reduces every firm's compliance effort. However, that one firm has a lower detection probability under regulator testing, as in the example, is necessary for regulator testing to strictly reduce every firm's compliance effort.

[6] To implement this in practice, government could give a regulator responsibility and budget only to verify reported violations.

[7] To be precise, a sufficient condition for nonzero regulator testing (setting $t_{Ri} > 0$ for at least one $i \in \mathcal{N}$ instead of $t_{Ri} = 0$ for $i \in \mathcal{N}$) to strictly reduce social welfare is (7) in Proposition 2(a), which ensures $\tau_i > 0$ for at least one $i \in \mathcal{N}$; $t_{Ri} \leq \tau_i$ for $i \in \mathcal{N}$; and $(\partial/\partial t_{Ri}) d_i(\mathbf{t}_i) < (\partial/\partial t_{ji}) d_i(\mathbf{t}_i)$ for all $i, j \in \mathcal{N}$ with $i \neq j$ and $\mathbf{t}_i \in \mathcal{R}^+_N$, meaning that the regulator is less efficient than the firms in testing.

[8] As shown in the derivation of equilibrium prices in the appendix, a market allocates higher-quality products to consumers with higher willingness-to-pay-for-quality α. If firm i knocks out a higher-quality competitor j (one with $u_j > u_i$), then consumers with higher α buy firm i's product, so firm i's price p_i increases by an amount proportional to firm i's own quality u_i. The market price p_i for firm i's product depends on the quality levels of firms with lower quality (not the quality levels of firms with higher quality) because the price of each product is determined by its marginal consumer's indifference between purchasing that product or the next lower-quality product (or no product). If firm i knocks out a lower-quality competitor j (one with $u_j < u_i$), the marginal consumer for firm i's product remains the same but her alternative is worse by an amount proportional to u_j, the quality of the product knocked out of the market, so the price she is willing to pay for product i increases by an amount proportional to u_j.

References

Anand KS, Girotra K (2007) The strategic perils of delayed differentiation. *Management Sci.* 53(5):697–712.

Austen-Smith D, Feddersen TJ (2008) Public disclosure, private revelation, or silence: Whistleblowing incentives and managerial policy. Working paper, Northwestern University, Evanston, IL.

Babich V, Tang CS (2012) Managing opportunistic supplier adulteration: Deferred payments, inspection, and combined mechanisms. *Manufacturing Service Oper. Management* 14(2):301–314.

Baiman S, Fischer PE, Rajan MV (2000) Information, contracting, and quality costs. *Management Sci.* 46(6):776–789.

Balachandran KR, Radhakrishnan S (2005) Quality implications of warranties in a supply chain. *Management Sci.* 51(8):1266–1277.

Bigoni M, Fridolfsson S-O, Le Coq C, Spagnolo G (2012) Fines, leniency, and rewards in antitrust. *RAND J. Econom.* 43(2): 368–390.

Boyer M, Lewis TR, Liu WL (2000) Setting standards for credible compliance and law enforcement. *Canadian J. Econom.* 33(2): 319–340.

Brown R (2012) Interview with Richard Brown, Research Scientist, Environmental Energy Technologies Division, Lawrence Berkeley National Laboratory, August 8.

Bruschia R (2008) Personal interview and email exchange with Robert Bruschia, research scientist, California EPA's Department of Toxic Substances Control, August 1–25.

Chod J, Rudi N (2005) Resource flexibility with responsive pricing. *Oper. Res.* 53(3):532–548.

Cohen M (1999) Monitoring and enforcement of environmental policy. Tietenberg T, Folmer H, eds. *The International Yearbook of Environmental and Resource Economics* 1999/2000 (Edward Elgar Publishers, Northampton, MA), 1–59.

Croft SL, Strongman J (2004) *The Revised General Product Safety Directive: A Practical Look. Global Counsel Dispute Resolution Handbook* (Thompson Reuters Practical Law, Eagan, MN).

Darby MR, Karni E (1973) Free competition and the optimal amount of fraud. *J. Law Econom.* 16(1):67–88.

Day G (2007) *The Market Driven Organization: Understanding, Attracting, and Keeping Valuable Customers* (Free Press, New York).

Department of Energy (2010) Press release: EPA, DOE announce new steps to strengthen Energy Star, March 19.

Dohmen F, Hawranek D (2017) The Cartel: Collusion between Germany's biggest carmakers. *Der Spiegel* (July 22):12.

European Commission (2012) Rapid Alert System for dangerous non-food products. Last accessed July 1, 2018, http://ec.europa.eu/consumers/safety/rapex/stats_reports_en.htm.

European Commission (2015) Rapid Alert System—Most recent alerts. Last accessed July 1, 2018, http://ec.europa.eu/consumers/consumers_safety/safety_products/rapex/reports/docs/rapex_annual_report_2015_en.pdf.

Ewing J (2017) Inside VW's campaign of trickery. *New York Times* (May 7):BU1.

Feddersen TJ, Gilligan TW (2001) Saints and markets: Activists and the supply of credence goods. *J. Econom. Management Strategy* 10(1):149–171.

Gaffigan M (2007) Energy efficiency: Opportunities exist for federal agencies to better inform household consumers—Statement of Mark Gaffigan, Acting Director, Natural Resources and Environment. Report GAO-07-116, General Accountability Office, Washington, DC.

GAO (2010) Energy Star Program: Covert testing shows the Energy Star Program certification process is vulnerable to fraud and abuse. Report GAO-10-470, General Accountability Office, Washington, DC.

Green Supply Line (2006) RoHS police are busy at work. *United Business Media* (November 27), http://greensupplyline.com/showArticle.jhtml?articleID=196500146.

Hess R (2006) Ensuring compliance: A content perspective. *Presentation at Stanford Global Supply Chain Management Forum, Stanford University, April 19*.

Heyes A, Martin S (2017) Social labeling by competing NGOs: A model with multiple issues and entry. *Management Sci.* 63(6): 1800–1813.

Hwang I, Radhakrishnan S, Su L (2006) Vendor certification and appraisal: Implications for supplier quality. *Management Sci.* 52(10):1472–1482.

RIGHTSLINK

Kapoor A (2012) Focus on product identification: What applies now that the new Product Safety Act has come into effect? *PSI J. Online*. Accessed July 1, 2012, http://psi-network.org/interview_mit_rechtsanwalt_dr_arun_kapoor_1-2374.html.

Ko Y, Simons KL (2016) The adoption of energy efficient light bulbs and information diffusion of multidimensional product characteristics: A text-mining analysis using Amazon product webpages. Working paper, Rensselaer Polytechnic Institute, Troy, NY.

Lawrence D, Mehrotra K, Rauwald C, Behrmann E, Beene R (2017) Will Bosch choke on VW's exhaust? *Bloomberg Businessweek* (August 14):12–14.

Li X, Peeters R (2017) Rivalry information acquisition and disclosure. *J. Econom. Management Strategy* 26(3):610–623.

Mas-Colell A, Whinston MD, Green JR (1995) *Microeconomic Theory* (Oxford University Press, New York).

Mason CF (2011) Eco-labeling and market equilibria with noisy certification tests. *Environment. Resource Econom.* 48(4):537–560.

McCluskey JJ, Loureiro ML (2005) Reputation and production standards. *J. Agricultural and Resource Econom.* 30(1):1–11.

Mookherjee D, Png IPL (1992) Monitoring vis-á-vis investigation in enforcement of law. *Amer. Econom. Rev.* 82(3):556–565.

Motta M (1993) Endogenous quality choice: Price vs. quantity competition. *J. Indust. Econom.* 41(2):113–131.

Nord N (2010) Congressional Testimony by Nancy Noord, U.S. Consumer Product Safety Comissioner. Last accessed July 1, 2018, http://www.cpsc.gov/s3fs-public/nord03042010.pdf.

Overfelt M (2006) Total recall. *Fortune Small Bus.* 16(1):88–93.

Polinsky AM (1980) Private versus public enforcement of fines. *J. Legal Stud.* 9(1):105–127.

Porteus EL, Angelus A (1997) Opportunities for improved statistical process control. *Management Sci.* 43(9):1214–1228.

Roe B, Sheldon I (2007) Credence good labeling: The efficiency and distributional implications of several policy approaches. *Amer. J. Agricultural Econom.* 89(4):1020–1033.

Rosner S (2010) Energy Star certification now tougher. *Mother Earth News* (Aug./Sept).

Ross D (2007) Personal interview with Dale Ross, former Mars, Inc. executive. Dec. (13).

Saphores J-DM, Nixon H, Ogunseitan OA, Shapiro AA (2007) California households' willingness to pay for "green" electronics. *J. Environment. Planning and Management* 50(1):113–133.

Shah JB, Sullivan L (2002) Firms warned to brace for environmental fallout—New laws threaten to disrupt the unprepared. *Electronic Buyer's News* (September 30):1–2.

Smith C (2008) Personal interview with Chris Smith, technical manager of the United Kingdom's RoHS Enforcement Authority and secretary of the European RoHS Enforcement Network, September 4.

Spagnolo G (2008) Leniency and whistleblowers in antitrust. Buccirossi P, ed. *Handbook of Antitrust Economics*, Chap. 7 (MIT Press, Cambridge, MA), 259–303.

Story L (2008) Can Burt's Bees turn Clorox green? *New York Times* (January 6):C1.

Struck H (2008) Dr. Bronner's organic discord. *Forbes* (May 1), forbes.com/2008/05/01/bronner-organic-soap-faces-markets-cx_hs_0501autofacescan01.html.

Swinney R, Cachon G, Netessine S (2011) Capacity investment timing by start-ups and established firms in new markets. *Management Sci.* 57(4):763–777.

Ting MM (2008) Whistleblowing. *Amer. Political Sci. Rev.* 102(2):249–267.

Vestel LB (2009) Energy Star gets tough on LG Electronics. *New York Times* (December 30), https://green.blogs.nytimes.com/2009/12/30/energy-star-gets-tough-on-lg-electronics/.

Wiel S, McMahon JE (2005) *Energy-Efficiency Labels and Standards: A Guidebook for Appliances, Equipment and Lighting*, 2nd ed. (Collaborative Labeling and Appliance Standards Program (CLASP), Washington, DC).

Winfree JA, McCluskey JJ (2005) Collective reputation and quality. *Amer. J. Agricultural Econom.* 87(1):206–213.

Zhu K, Zhang RQ, Tsung F (2007) Pushing quality improvement along supply chains. *Management Sci.* 53(3):421–436.

RIGHTSLINK